Names:
Math 258 — Using Power Series — December 2, 2015

$$e^u = \sum_{n=0}^{\infty} \frac{u^n}{n!} \text{ for all } u$$

1. Recall that $\sinh x = \frac{1}{2}\left(e^x - e^{-x}\right)$ and $\cosh x = \frac{1}{2}\left(e^x + e^{-x}\right)$.

a) Use the power series expansions of e^x and e^{-x} to find power series expansions of $\sinh x$ and $\cosh x$.

b) Use your power series expansion of $\sinh x$ to help evaluate the limit $\lim_{x\to 0} \frac{\sinh x}{x}$ (without using l'Hospital's rule).

c) Use your power series expansion of $\cosh x$ to help evaluate the limit $\lim_{x\to 0} \frac{1 - \cosh x}{x}$ (without using l'Hospital's rule).

$$(1+u)^k = 1 + ku + \frac{k(k-1)}{2!}u^2 + \frac{k(k-1)(k-2)}{3!}u^3 + \frac{k(k-1)(k-2)(k-3)}{4!}u^4 + \dots \text{ if } |u| < 1$$

2. Recall that $\frac{d}{dx}\left(\sinh^{-1} x\right) = \frac{1}{\sqrt{1+x^2}}$.

a) Use the binomial series to find a power series expansion for $\frac{1}{\sqrt{1+x^2}}$ (there is no need to find a nice pattern–just write out at least the first 4 terms of the series).

b) Anti-differentiate your series to find a power series for $\sinh^{-1} x$ (again, there is no need to find a nice pattern).

c) Use your power series to find and approximation to $\sinh^{-1}\left(\frac{1}{2}\right)$.