

TABLE 8.1 Basic integration formulas

1. $\int k \, dx = kx + C$ (any number k)	12. $\int \tan x \, dx = \ln \sec x + C$
2. $\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$ ($n \neq -1$)	13. $\int \cot x \, dx = \ln \sin x + C$
3. $\int \frac{dx}{x} = \ln x + C$	14. $\int \sec x \, dx = \ln \sec x + \tan x + C$
4. $\int e^x \, dx = e^x + C$	15. $\int \csc x \, dx = -\ln \csc x + \cot x + C$
5. $\int a^x \, dx = \frac{a^x}{\ln a} + C$ ($a > 0, a \neq 1$)	16. $\int \sinh x \, dx = \cosh x + C$
6. $\int \sin x \, dx = -\cos x + C$	17. $\int \cosh x \, dx = \sinh x + C$
7. $\int \cos x \, dx = \sin x + C$	18. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$
8. $\int \sec^2 x \, dx = \tan x + C$	19. $\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$
8. $\int \csc^2 x \, dx = -\cot x + C$	20. $\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a} \sec^{-1}\left \frac{x}{a}\right + C$
10. $\int \sec x \tan x \, dx = \sec x + C$	21. $\int \frac{dx}{\sqrt{a^2 + x^2}} = \sinh^{-1}\left(\frac{x}{a}\right) + C$ ($a > 0$)
11. $\int \csc x \cot x \, dx = -\csc x + C$	22. $\int \frac{dx}{\sqrt{x^2 - a^2}} = \cosh^{-1}\left(\frac{x}{a}\right) + C$ ($x > a > 0$)

EXAMPLE 2 Complete the square to evaluate

$$\int \frac{dx}{\sqrt{8x - x^2}}$$

Solution We complete the square to simplify the denominator:

$$\begin{aligned} 8x - x^2 &= -(x^2 - 8x) = -(x^2 - 8x + 16 - 16) \\ &= -(x^2 - 8x + 16) + 16 = 16 - (x - 4)^2. \end{aligned}$$

Then

$$\begin{aligned} \int \frac{dx}{\sqrt{8x - x^2}} &= \int \frac{dx}{\sqrt{16 - (x - 4)^2}} \\ &= \int \frac{du}{\sqrt{a^2 - u^2}} && a = 4, u = (x - 4), \\ &&& du = dx \\ &= \sin^{-1}\left(\frac{u}{a}\right) + C && \text{Table 8.1, Formula 18} \\ &= \sin^{-1}\left(\frac{x - 4}{4}\right) + C. \end{aligned}$$

