IMPROPER INTEGRALS

1. Evaluate the improper integrals (be sure to properly include a limit). Then write a general rule.

a) $\int_1^\infty \frac{1}{x^2}\, dx$

b) $\int_1^\infty \frac{1}{x^{1.5}}\, dx$

c) $\int_1^\infty \frac{1}{x}\, dx = \lim_{h\to\infty} \int_1^h \frac{1}{x}\, dx = \lim_{h\to\infty} \ln|h| = \infty$ (the improper integral diverges).

d) $\int_1^\infty \frac{1}{x^{0.75}}\, dx$

e) $\int_1^\infty \frac{1}{x^{0.5}}\, dx$

f) $\int_1^\infty \frac{1}{x^p}\, dx$ converges if ______________________________ and diverges otherwise.

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2. Let R be the region under the curve $y = 1/x$ over the interval $[1, \infty)$. Note that part c of problem 1 showed that R does not have finite area.

a) Find the volume of the solid formed by revolving R around the x-axis.

b) Set up an integral for the surface area of the same solid and determine if it converges or diverges. Hint: if $f(x) \geq g(x)$, then $\int_1^\infty f(x)\, dx \geq \int_1^\infty g(x)\, dx$.