INFINITE SERIES

Theorem. *If* $|r| < 1$, *then* $\displaystyle\sum_{n=1}^{\infty} r^n = \frac{r}{1-r}$.

1. Find the sum of the series:

a) $\sum_{n=1}^{\infty} \left(-\frac{3}{4}\right)^n$

b) $\sum_{n=3}^{\infty} \left(-\frac{3}{4}\right)^n$

c) $\sum_{n=0}^{\infty} r^n$ (where $|r| < 1$)

2. The decimal $0.999\ldots$ can be expressed as an infinite series $\displaystyle\sum_{n=1}^{\infty} \frac{9}{10^n}$.

a) Verify that the series $\displaystyle\sum_{n=1}^{\infty} \frac{9}{10^n}$ does, in fact, sum to 1.

b) Use the same approach to find a rational expression for the number $0.777\ldots$

c) Use the same approach to find a rational expression for the number $0.272727\ldots$

Date: March 8, 2019.

3. Some series are called telescoping series because middle terms of the partial sums cancel out, giving a simple formula for S_n.

a) Find a simple formula for the partial sums of $\sum_{k=1}^{\infty}\left(\frac{1}{k+1}-\frac{1}{k+2}\right)$.

b) Take the limit to find the sum of the series.

c) Use a similar approach to find the sum of $\sum_{k=1}^{\infty}\left(\frac{1}{k(k+1)}\right)$. Hint: start with a partial fractions decomposition.