TAYLOR POLYNOMIALS

Definition. The degree n Taylor polynomial for $f(x)$ centered at a is

$$p_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x-a)^k = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \dots \frac{f^{(n)}(a)}{n!}(x-a)^n.$$

Theorem. *If* $f^{(n+1)}(c) \leq M$ *for all* c *between* a *and* x*, then*

$$|R_n(x)| = |f(x) - p_n(x)| \leq \frac{M}{(n+1)!} |x-a|^{n+1}.$$

1. Let $f(x) = \cos x$.

a) Find $p_7(x)$, the degree 7 Taylor polynomial for f centered at 0.

b) Estimate $\cos(1)$ by calculating $p_7(1)$.

c) Use the theorem above to find an upper bound for the error of your estimate.

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2. Let $f(x) = e^x$.

a) Find $p_4(x)$, the degree 4 Taylor polynomial for f centered at 0.

b) Use p_4 to estimate $\sqrt{e}$.

c) Use the theorem above to find an upper bound for the error of your estimate.

d) Use a calculator to find the actual error of your estimate.

e) Find an upper bound for the error of $p_n(1/2)$ as an estimate for $\sqrt{e}$ and take the limit as $n \to \infty$. Interpret the result.

3. Let $f(x) = \frac{1}{1-x}$.

a) Find $p_4(x)$, the degree 4 Taylor polynomial for f centered at 0.

b) Find $q_4(x)$, the degree 4 Taylor polynomial for f centered at -1.