POWER SERIES

1. Use the ratio test to determine the radius of convergence and interval of convergence of the the power series $\sum_{k=1}^{\infty} \frac{(x-1)^k}{k}$

2. Use the root test to determine the radius of convergence of the the power series $\sum_{k=1}^{\infty} \frac{(x-1)^k}{k^k}$

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Definition. The Taylor series for $f(x)$ centered at a is

$$\sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!}(x-a)^k = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f^{(3)}(a)}{3!}(x-a)^3 \dots$$

(compare with the Taylor polynomial).

3. Find the first 4 terms of the Taylor series for $f(x) = \tan^{-1} x$ centered at 0 by differentiating f (this is the same as the degree 3 Taylor polynomial).

4. You were probably happy to stop taking derivatives in the last problem, but you also probably didn't get enough of the Taylor series to find a pattern. Instead, find the Taylor series by making use of the fact that $\frac{d}{dx}[\tan^{-1} x] = \frac{1}{1+x^2}$ and if $|r| < 1$, then $\frac{1}{1-r} = \sum_{k=0}^{\infty} r^k$.

5 (Optional)**.** On the last worksheet you found the degree 7 Taylor polynomial for $\cos x$ centered at 0. Continuing the pattern gives a Taylor series:

$$\cos x = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k}}{(2k)!}$$

a) Use this power series to find a power series for $\sin x$.

b) Use the power series for $\sin x$ to show that $\lim_{x \to 0} \frac{\sin x}{x} = 1$.

c) Your argument for the last part only works if the radius of convergence for the series you found is greater than 0. Find the radius of convergence.