USING POWER SERIES

Method. To find a power series representation of a function $f(x)$:

1) Work from a known series and differentiate, integrate, substitute, etc. Some known series:

$$\frac{1}{1-u} = \sum_{k=0}^{\infty} u^k = 1 + u + u^2 + u^3 + u^4 + \dots \text{ if } |u| < 1$$

$$e^u = \sum_{k=0}^{\infty} \frac{u^k}{k!} = 1 + u + \frac{u^2}{2} + \frac{u^3}{3!} + \frac{u^4}{4!} + \dots \text{ for all } u$$

$$\sin u = \sum_{k=0}^{\infty} (-1)^k \frac{u^{2k+1}}{(2k+1)!} = u - \frac{u^3}{3!} + \frac{u^5}{5!} - \frac{u^7}{7!} + \dots \text{ for all } u$$

2) If method 1 fails, differentiate to find the the Taylor series centered at a: $f(x) = \sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!}(x-a)^k$.

1. Use power series to evaluate the limits:

a) $\lim_{x \to 0} \frac{\sin x}{x}$

b) $\lim_{x \to \infty} x \sin(1/x)$

c) $\lim_{x \to \infty} x \sin\left(1/x^2\right)$

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2. Use power series to evaluate the integrals (find at least 4 nonzero terms):

a) $\displaystyle\int_0^1 e^{-x^2}\,dx$

b) $\displaystyle\int_0^{\frac{1}{2}} \frac{1}{1-x^3}\,dx$

3. The **Fresnel integrals** are $S(x) = \displaystyle\int_0^x \sin(t^2)\,dt$ and $C(x) = \displaystyle\int_0^x \cos(t^2)\,dt$. Find at least 4 nonzero terms of power series representations of $S(x)$ and $C(x)$.

Challenge. Evaluate the integral: $\displaystyle\int_{-1}^0 \frac{\ln(1-x)}{x}\,dx$