

POLAR CALCULUS

Method. The curve $r = f(\theta)$ can be parameterized as $x = f(\theta) \cos \theta$, $y = f(\theta) \sin \theta$. Then $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$.

The arc length of the curve is given by $L = \int_{\alpha}^{\beta} \sqrt{[f(\theta)]^2 + [f'(\theta)]^2} d\theta$. The area inside the curve is

$$A = \int_{\alpha}^{\beta} \frac{1}{2} [f(\theta)]^2 d\theta.$$

1. Find an equation for the tangent line to the cardioid $r = 1 + \cos \theta$ when $\theta = \frac{\pi}{3}$.

2. Calculate the arc length of the cardioid $r = 1 + \cos \theta$.

- 3.** Find the area outside $r = 1 + \cos \theta$ and inside $r = 3 \cos \theta$ (link to Desmos).
- 4.** Find equations for the tangent lines to the cardioid $r = 1 + 2 \cos \theta$ at $\theta = \frac{2\pi}{3}$ and at $\theta = \frac{4\pi}{3}$. Plot the cardioid and both lines (link to Desmos). What's up with that?