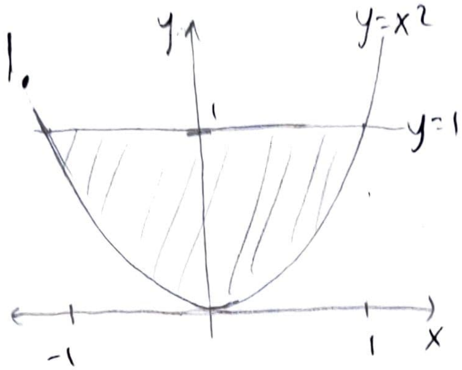
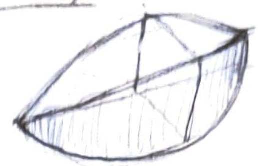
258 F20 Exam 1 (Chapter 6)

1.



a)



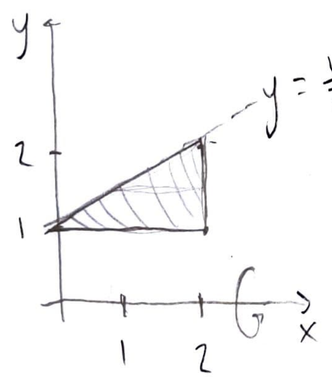
$$V = \int_{-1}^{1} (1-x^2)^2 \, dx$$

b) $V = \int_0^1 \frac{1}{2}\pi \left[\sqrt{y}\right]^2 dy$

w/o left half $V = \int_0^1 \frac{1}{2}\pi \left[\frac{\sqrt{y}}{2}\right]^2 dy$

$= \int_0^1 \frac{\pi}{8} y \, dy$

2.



$y = \frac{1}{2}x + 1$ or $x = 2y - 2$

a) $V = \int_0^2 \pi \left[\left(\frac{1}{2}x+1\right)^2 - 1^2\right] dx$ washers

b) $V = \int_1^2 2\pi y \left[2 - (2y-2)\right] dy$ shells

3. Shells: $V = \int_0^{\sqrt{\pi}} 2\pi x \sin(x^2)\,dx$ $\quad u = x^2 \quad du = 2x\,dx$

$$= \int_0^{\pi} \pi \sin u \, du$$

$$= -\pi \cos u \Big|_0^{\pi}$$

$$= -\pi \cos(\pi) + \pi \cos(0) = 2\pi$$

4. Washers/disks; use $x = \sqrt{1-y^4}$ (the right half)

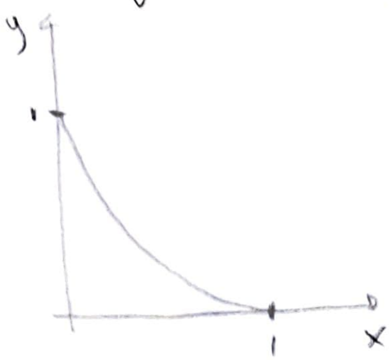
$$V = 2\int_0^1 \pi (1-y^4)\,dy = 2\pi\left[y - \tfrac{1}{5}y^5\right]_0^1 = 2\pi\left(1-\tfrac{1}{5}\right) = \frac{8\pi}{5}$$

Shells; use $y = (1-x^2)^{1/4}$ (the top half)

$$V = 2\int_0^1 2\pi x (1-x^2)^{1/4}\,dx \qquad u = 1-x^2 \quad du = -2x\,dx$$

$$= 2\pi \int_0^1 u^{1/4}\,du = 2\pi\left[\tfrac{4}{5}u^{5/4}\right]_0^1 = \frac{8\pi}{5}$$

5. $\sqrt{x}+\sqrt{y}=1 \rightarrow y=(1-\sqrt{x})^2 \rightsquigarrow \frac{dy}{dx} = 2(1-\sqrt{x})\left(-\frac{1}{2}\right)x^{-\frac{1}{2}} = \frac{\sqrt{x}-1}{\sqrt{x}}$



$= 1-2\sqrt{x}+x \rightsquigarrow \frac{dy}{dx} = 1-\frac{1}{\sqrt{x}}$

$\left(\frac{dy}{dx}\right)^2 = \left(1-\frac{1}{\sqrt{x}}\right)^2 = 1-\frac{2}{\sqrt{x}}+\frac{1}{x}$

$$L = \int_0^1 \sqrt{1+\left(1-\frac{1}{\sqrt{x}}\right)^2}\,dx = \int_0^1 \sqrt{2-\frac{2}{\sqrt{x}}+\frac{1}{x}}\,dx$$

6. $x = y^3/6$, $0 \le y \le 2$, rotate about y-axis

$\frac{dx}{dy} = \frac{y^2}{2}$

$$SA = \int_0^2 2\pi\left(\frac{y^3}{6}\right)\sqrt{1+y^4/4}\,dy$$

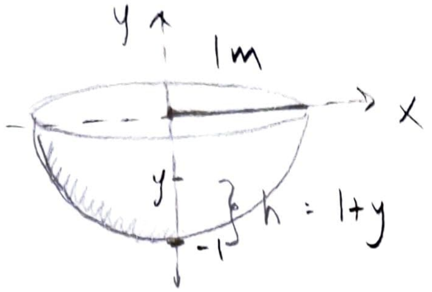
$u = 1+\frac{y^4}{4}$

$du = y^3\,dy$

$$= \int_1^5 \frac{\pi}{3}\sqrt{u}\,du$$

$$= \frac{\pi}{3}\left(\frac{2}{3}\right)u^{3/2}\Big|_1^5$$

$$= \frac{2\pi}{9}\left[5^{3/2}-1\right]$$

7. 

$x^2+y^2=1 \rightarrow x=\pm\sqrt{1-y^2}$

$$W = \int_{-1}^{0} (1+y)\rho g \pi \left(\sqrt{1-y^2}\right)^2 dy$$

$$= \pi \rho g \int_{-1}^{0} (1+y)(1-y^2)\, dy$$

$$= \pi \rho g \int_{-1}^{0} 1+y-y^2-y^3\, dy$$

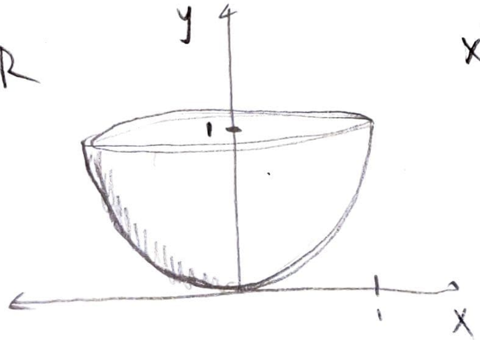
$$= \pi \rho g \left[y+\frac{1}{2}y^2-\frac{1}{3}y^3-\frac{1}{4}y^4\right]_{-1}^{0}$$

$$= \pi \rho g \left[-\left(-1+\frac{1}{2}+\frac{1}{3}-\frac{1}{4}\right)\right]$$

$$= \pi \rho g \left[\frac{12-6-4+3}{12}\right]$$

$$= \frac{5\pi}{12}\rho g$$

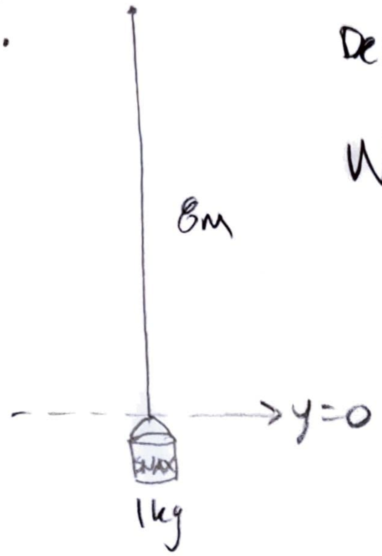
OR



$x^2+(y-1)^2=1 \qquad x=\pm\sqrt{1-(y-1)^2}$

$$W = \int_{0}^{1} y \rho g \pi \left(\sqrt{1-(y-1)^2}\right) dy$$

8.



Density of the rope: $2kg/8m = \frac{1}{4}\frac{kg}{m}$

$$W = \int_0^8 \left[(8-y)\tfrac{1}{4} + 2\right] 9.8 \, dy$$

$$= 9.8 \int_0^8 4 - \frac{y}{4} \, dy$$

$$= 9.8 \left[4y - \frac{y^2}{8}\right]_0^8$$

$$= 9.8 \left[32 - 8\right]$$

$$= 9.8(24) \text{ J}$$