CHAPTER 6 EXAM

Instructions. Solve the following 8 problems and write your solutions clearly, showing all work necessary for your solution. A correct solution without supporting work may receive little or no credit. Clearer work will generally earn more points than work that is hard to read. You are allowed to use course resources: books, class notes, worksheet solutions, course videos, etc. I've given you some interesting curves and you are allowed to use Desmos to help visualize them. However, you should **evaluate all integrals without a calculator or computer** (in part because you'll need to show your work). Solutions should not be simplified (I don't expect you to calculate $13600(9.8)$ or $17^{3/2}$, but those are perfectly good numbers as written and will work for your solutions). **You may not collaborate with other people**; everything you turn in should be your own work an no one else's.

1. The base of a solid is the region enclosed between the curves $y = x^2$ and $y = 1$. Set up integrals giving the volume of the solid with the following cross-sections. **You do not need to evaluate the integrals.**

a) Cross-sections perpendicular to the x-axis are squares

b) Cross-sections perpendicular to the y-axis are semi-circles (with flat side spanning the base region)

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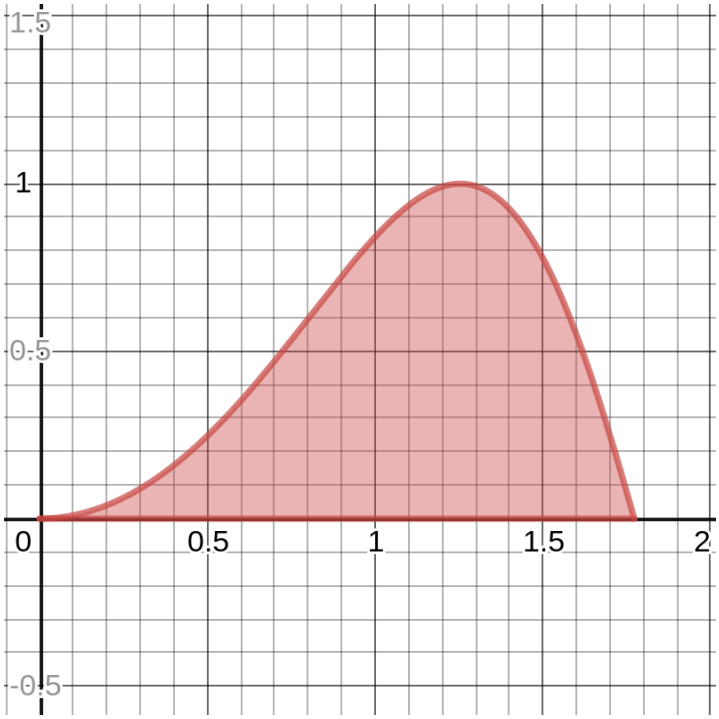
2. Consider the solid formed by rotating the triangle with vertices $(0, 1)$, $(2, 1)$, and $(2, 2)$ about the x-axis. The goal of this problem is set up integrals giving the volume of the solid using both the disk/washer method and the shell method. **You do not need to evaluate the integrals** (though doing so may help you check your solutions).

a) Find a dx integral for the volume of the solid

b) Find a dy integral for the volume of the solid

3. Find the volume of the solid formed by rotating the region between the x-axis and $y = \sin(x^2)$ with $0 \leq x \leq \sqrt{\pi}$ (the region is shown in the plot below) about the y-axis. Your answer should be a number (you must evaluate the integral, but you don't need to simplify your solution).



4. Find the volume of the solid formed by rotating the curve $x^2 + y^4 = 1$ about the y-axis. Your answer should be a number (you must evaluate the integral, but you don't need to simplify your solution).

5. The curve $\sqrt{x} + \sqrt{y} = 1$ is defined for $0 \leq x \leq 1$ and $0 \leq y \leq 1$. Set up but **do not evaluate** an integral giving the arc length of the curve.

6. Find the area of the surface formed by rotating the curve $x = \dfrac{y^3}{6}$ for $0 \leq y \leq 2$ about the y-axis. Your answer should be a number (you must evaluate the integral, but you don't need to simplify your solution).

7. A tank has the shape of a hemisphere of radius 1 m with the flat side on top. How much work is required to fill the tank with mercury by pumping it in from the bottom? For the purposes of this problem Mercury has a density of $\rho = 13,500$ kg/m^3 (this is pretty close to the actual density of Mercury at 35° C according to `https://srdata.nist.gov/solubility/IUPAC/SDS-29/SDS-29-pages_237.pdf`) and the acceleration of gravity is $g = 9.8$ m/s^2. You may use the symbols ρ and g instead of the corresponding numbers if you wish, including in your final answer. Other than ρ and g, your answer should be a number (you must evaluate the integral, but you don't need to simplify your solution).

8. A 1 kg bucket of snacks hangs from the end of an 8 m long rope. The mass of the rope (without the bucket) is 2 kg. How much work is required to pull the snacks up to your treehouse? The acceleration of gravity is $g = 9.8$ m/s^2 and, as in the previous problem, you may use g instead of 9.8 in your solution. Other than ρ and g, your answer should be a number (you must evaluate the integral, but you don't need to simplify your solution).