258 F20 Ch. 8 Exam Solutions (Exam 2)

1. $\int \tan x \sec^3 x\,dx = \int \sec^2 x\,(\tan x \sec x)\,dx$

$u = \sec x$
$du = \tan x \sec x\,dx$

$= \int u^2\,du$

$= \frac{1}{3}u^3 + C$

$= \frac{1}{3}\sec^3 x + C$

2. $\int 6x^2 \sin^2(x^3)\,dx = \int 2\sin^2 u\,du$

$u = x^3$
$du = 3x^2\,dx$

$= \int 1 - \cos 2u\,du$

$= u - \frac{1}{2}\sin 2u + C$

$= x^3 - \frac{1}{2}\sin(2x^3) + C$

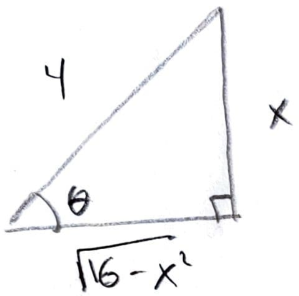
$= x^3 - \sin x^3 \cos x^3 + C$

3. $\displaystyle\int \frac{1}{(16-x^2)^{3/2}}\,dx = \int \frac{4\cos\theta}{(16-16\sin^2\theta)^{3/2}}\,d\theta$

$x = 4\sin\theta$

$dx = 4\cos\theta\,d\theta$

$$= \int \frac{4\cos\theta}{(4\cos\theta)^3}\,d\theta$$



$\sin\theta = \frac{x}{4}$

$$= \int \frac{1}{16\cos^2\theta}\,d\theta$$

$$= \int \frac{1}{16}\sec^2\theta\,d\theta$$

$$= \frac{1}{16}\tan\theta + C$$

$$= \frac{1}{16}\left(\frac{x}{\sqrt{16-x^2}}\right) + C$$

4. $\displaystyle\int \frac{x^2 - x - 3}{(x-1)(x^2+2)}\,dx = \int \frac{-1}{x-1} + \frac{2x}{x^2+2} + \frac{1}{x^2+2}\,dx$

$\displaystyle = -\ln|x-1| + \ln|x^2+2| + \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + C$

Partial fractions

$$\frac{x^2 - x - 3}{(x-1)(x^2+2)} = \frac{A}{x-1} + \frac{Bx + C}{x^2+2} = \frac{A(x^2+2) + (Bx+C)(x-1)}{(x-1)(x^2+2)}$$

$$\boxed{x^2 - x - 3 = A(x^2+2) + (Bx+C)(x-1)}$$

$$= (A+B)x^2 + (-B+C)x + 2A - C$$

$x=1$: $\quad 1-1-3 = A(1+2) \implies A = -1$

$A + B = 1$ so $B = 2$

$2A - C = -3$ so $-2 - C = -3$ and $C = 1$

5. $\int x^{258} \ln x \, dx = \frac{1}{259} x^{259} \ln x - \int \frac{1}{259} x^{258} \, dx$

IBP: $u = \ln x \quad du = \frac{1}{x} dx$

$dv = x^{258} dx \quad v = \frac{1}{259} x^{259} \quad dx$

$= \frac{1}{259} x^{259} \ln x - \frac{1}{(259)^2} x^{259} + C$

6. $\int_0^1 x^3 e^{x^2} dx = \frac{x^2}{2} e^{x^2} \Big|_0^1 - \int_0^1 x e^{x^2} dx$

IBP $u = x^2 \quad du = 2x \, dx$

$dv = x e^{x^2} dx \quad v = \frac{1}{2} e^{x^2}$

$= \left[\frac{1}{2} e - 0\right] - \left[\frac{1}{2} e^{x^2}\right]_0^1$

$= \frac{1}{2} e - \left[\frac{1}{2} e - \frac{1}{2}\right]$

$= \frac{1}{2}$

* 7. $\displaystyle\int_0^2 \frac{1}{(2-x)^5}\,dx = \int_2^0 -\frac{1}{u^5}\,du$

$u = 2-x$ $\qquad$ $\displaystyle = \int_0^2 \frac{1}{u^5}\,du$ $\qquad$ Divergent

$du = -dx$

OR $\displaystyle\int_0^2 \frac{1}{(2-x)^5}\,dx = \lim_{b\to 2^-} \int_0^b \frac{1}{(2-x)^5}\,dx$

$$= \lim_{b\to 2^-} \left.\frac{1}{4(2-x)^4}\right|_0^b$$

$$= \lim_{b\to 2^-} \frac{1}{4\underbrace{(2-b)^4}_{\to 0^+}} - \frac{1}{2^6}$$

$$= \infty$$

8. $\int_7^\infty \frac{2x}{1+x^5}\,dx$

For $x \geq 7$: $0 \leq \frac{2x}{1+x^5} \leq \frac{2x}{x^5} = \frac{2}{x^4}$

and $\int_7^\infty \frac{2}{x^4}\,dx$ converges.

therefore the original integral converges by comparison.

9. $\int_1^\infty \frac{1}{1-x+2x^3}\,dx$

Limit comparison with $\frac{1}{x^3}$ $\left(\int_1^\infty \frac{1}{x^3}\,dx \text{ converges} \right)$

$$\lim_{x\to\infty} \frac{1/x^3}{1/(1-x+2x^3)} = \lim_{x\to\infty} \frac{1-x+2x^3}{x^3}$$

$$= \lim_{x\to\infty} \frac{1}{x^3} - \frac{1}{x^2} + 2$$

$$= 2$$

therefore the original integral converges.