258 F20 Solutions to the sequences summary problems 10/15

a) $\lim_{n\to\infty} \frac{n+(-1)^n}{n} = 1$

$\frac{n+(-1)^n}{n} = 1 + \frac{(-1)^n}{n}$ and $1-\frac{1}{n} \le 1+\frac{(-1)^n}{n} \le 1+\frac{1}{n}$

$\lim_{n\to\infty} 1-\frac{1}{n} = \lim_{n\to\infty} 1+\frac{1}{n} = 1$. By the squeeze theorem the limit of the original sequence is also 1.

b) $\lim_{n\to\infty} \frac{1-n^3}{100-4n^2} = \lim_{n\to\infty} \frac{\frac{1}{n^3}-1}{\frac{100}{n^3}-\frac{4}{n}} = -\infty$

c) $\lim_{n\to\infty} (-1)^n\left(1-\frac{1}{n}\right)$ DNE

when n is even the sequence is close to 1

when n is odd the sequence is close to -1

d) $0 \le \frac{\sin^2 n}{n^2} \le \frac{1}{n^2}$ and $\lim_{n\to\infty} 0 = \lim_{n\to\infty} \frac{1}{n^2} = 0$.

Therefore $\lim_{n\to\infty} \frac{\sin^2 n}{n^2} = 0$ by the squeeze theorem.

e) $\frac{\ln(n)}{\ln(2n)} = \frac{\ln(n)}{\ln(2)+\ln(n)} = \frac{1}{\frac{\ln(2)}{\ln(n)}+1} \to 1$ as $n\to\infty$.

f) $\frac{(2n-1)!}{(2n+1)!} = \frac{(2n-1)!}{(2n+1)(2n)(2n-1)!} = \frac{1}{(2n+1)(2n)} \to 0$ as $n\to\infty$