

WS 1 Solutions

$$1. a) \int \frac{1}{3} x^5 dx = \frac{1}{18} x^6 + C$$

$$b) \int \frac{1}{x^3} dx = \int x^{-3} dx = -\frac{1}{2} x^{-2} + C = -\frac{1}{2x^2} + C$$

$$c) \int \sqrt{x} dx = \int x^{1/2} dx = \frac{2}{3} x^{3/2} + C$$

$$d) \int x^{-1} dx = \ln|x| + C$$

$$2. \int x^a dx = \begin{cases} \frac{1}{a+1} x^{a+1} + C & \text{if } a \neq -1 \\ \ln|x| + C & \text{if } a = -1 \end{cases}$$

3. chain rule.

$$a) \frac{2}{2x+1}$$

$$b) 2x(5)(x^2+1)^4 = 10x(x^2+1)^4$$

$$c) -3x^2 \sin(x^3)$$

$$4. \int \frac{6}{2x+1} dx = \int \frac{3}{u} du = 3 \ln|u| + C = 3 \ln|2x+1| + C$$

$u = 2x+1$
 $du = 2dx$

(substitution.)

$$5. a) \int x^3 \cos(x^4) dx = \int \frac{1}{4} \cos u du = \frac{1}{4} \sin u + C = \frac{1}{4} \sin(x^4) + C$$

$$u = x^4 \quad du = 4x^3 dx$$

$$b) \int x e^{(2x^2)} dx = \int \frac{1}{4} e^u du = \frac{1}{4} e^u + C = \frac{1}{4} e^{2x^2} + C$$

$$u = 2x^2 \quad du = 4x dx$$

challenge.

$$a) \int \frac{6x}{1-3x} dx = \int -3x \left(\frac{1}{1-3x} \right) \left(\frac{2}{3} \right) (-3) dx$$

$$u = 1-3x \quad du = -3dx$$

$$-3x = u-1$$

$$= \int (u-1) \left(\frac{1}{u} \right) \left(\frac{2}{3} \right) du$$

$$= \frac{2}{3} \int \left(1 - \frac{1}{u} \right) du$$

$$= \frac{2}{3} [u - \ln|u|] + C$$

$$= \frac{2}{3} [1-3x - \ln|1-3x|] + C$$

check by differentiating:

$$\frac{d}{dx} \left[\frac{2}{3} (1-3x - \ln|1-3x|) \right] = \frac{2}{3} \left(-3 - \frac{-3}{1-3x} \right)$$

$$= \frac{2}{3} \left(\frac{-3+9x+3}{1-3x} \right) = \frac{6x}{1-3x} \quad \checkmark$$

b) similar approach

6. a) $\int 9x^2 \sec^2(x^3) dx = \int 3 \sec^2 u du = 3 \tan u + C$
 $u = x^3 \quad du = 3x^2 dx$
 $= 3 \tan(x^3) + C$

b) Many correct answers here

c) Depends on calculator?

Wolfram is stumped by $\int 9x \sec^2(x^3) dx$