INTEGRATION REVIEW

Example. Evaluate $\int x^3\,dx$.

Solution. We must find the general form of an antiderivative of the function $f(x) = x^3$. This means finding a function $F(x)$ such that $F'(x) = x^3$. We know from the power rule for derivatives that $\frac{d}{dx}[x^4] = 4x^3$. It follows that $\frac{d}{dx}\left[\frac{1}{4}x^4\right] = x^3$ (multiplying both sides of the previous equation by $1/4$).

Therefore $\boxed{\int x^3\,dx = \frac{1}{4}x^4 + C}$.

1. Evaluate the following indefinite integrals. (You can always check your answers by differentiating).

a) $\int \frac{1}{3}x^5\,dx$

b) $\int \frac{1}{x^3}\,dx$

c) $\int \sqrt{x}\,dx$

d) $\int x^{-1}\,dx$

2. Fill in the blank to finish the statement: If a is a real number, then

$$\int x^a\,dx = \begin{cases} \qquad\qquad & \text{if} \\ & \text{if} \end{cases}$$

Date: September 2, 2020.

One theme of chapter 8 will be that differentiation rules (usually) have corresponding integration rules. You just stated an integration version of the power rule for derivatives. You should also know an integration version of another important differentiation rule.

3. What differentiation rule do you use in evaluating the following derivatives? (No need to evaluate these derivatives, though it may find it helpful to do so when you're working on problem 4).

a) $\frac{d}{dx}[\ln(2x+1)]$

b) $\frac{d}{dx}\left[(x^2+1)^5\right]$

c) $\frac{d}{dx}\left[\cos(x^3)\right]$

4. Evaluate the integral $\int \frac{6}{2x+1}\,dx$. What integration rule do you use in evaluating the integral?

5. Evaluate the following integrals.

a) $\int x^3 \cos(x^4)\,dx$

b) $\int x e^{(2x^2)}\,dx$

Rule of Thumb. If the integrand requires the chain rule to differentiate, then it requires substitution to integrate.

Sometimes creative applications of substitution (or other mathematical tricks) allow you to evaluate integrals.

Challenge. Evaluate the following integrals.

a) $\displaystyle\int \frac{6x}{1-3x}\,dx$

b) $\displaystyle\int 12x^3(x^2+1)^5\,dx$

6. Consider the integral $\displaystyle\int 9x^2\sec^2(x^3)\,dx$.

a) Evaluate the integral.

b) Change one number in the integral to make the integral impossible (for now).

c) Plug the new integral into Wolfram Alpha or your calculator. Bonus point if your integral also stumps the computer.