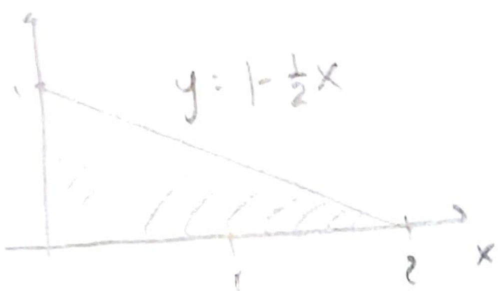
WS-02 Solutions

1.



a)

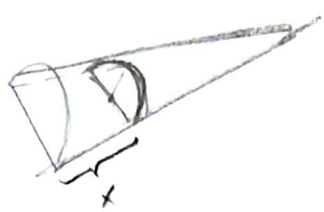
$$A(x) = \left(1 - \tfrac{1}{2}x\right)^2 = 1 - x + \tfrac{1}{4}x^2$$

$$V = \int_0^2 1 - x + \tfrac{1}{4}x^2 \, dx$$

$$= x - \tfrac{1}{2}x^2 + \tfrac{1}{12}x^3 \Big|_0^2$$

$$= 2 - 2 + \tfrac{8}{12} = \tfrac{2}{3}$$

b)

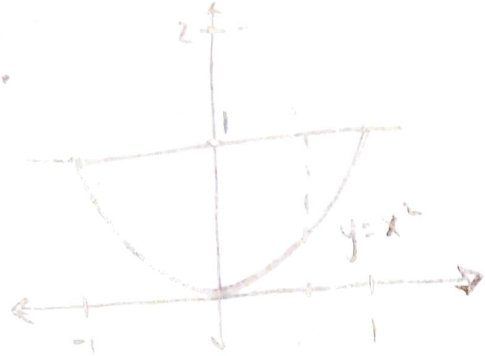


$$A(x) = \frac{\pi}{2}\Big[\underbrace{\tfrac{1}{2}\left(1 - \tfrac{1}{2}x\right)}_{\text{radius}}\Big]^2 = \frac{\pi}{8}\left(1 - x + \tfrac{1}{4}x^2\right)$$

$$V = \int_0^2 \frac{\pi}{8}\left(1 - x + \tfrac{1}{4}x^2\right) dx$$

$$= \frac{\pi}{8}\left(\frac{2}{3}\right) = \frac{\pi}{12}$$

2.



a) $R = 1 - x^2 \quad r = 0$

$$V = \int_{-1}^{1} \pi \left[(1-x^2)^2 \right] dx$$

$$= \int_{-1}^{1} \pi \left[1 - 2x^2 + x^4 \right] dx$$

$$= \pi \left[x - \frac{2}{3}x^3 + \frac{1}{5}x^5 \right]_{-1}^{1}$$

$$= \pi \left[1 - \frac{2}{3} + \frac{1}{5} - \left(-1 + \frac{2}{3} - \frac{1}{5}\right) \right]$$

$$= \pi \left[2 - \frac{4}{3} + \frac{2}{5} \right]$$

$$= \frac{\pi(30 - 20 + 6)}{15}$$

$$= \frac{16\pi}{15}$$

b) $R = 2 - x^2 \quad r = 1$

$$V = \int_{-1}^{1} \pi \left[(2-x^2)^2 - 1^2 \right] dx$$

$$= \int_{-1}^{1} \pi \left[3 - 4x^2 + x^4 \right] dx$$

$$= \pi \left[3x - \frac{4}{3}x^3 + \frac{1}{5}x^5 \right]_{-1}^{1}$$

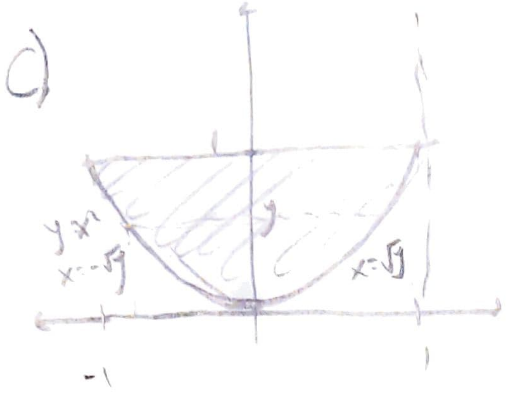
$$= \pi \left[3 - \frac{4}{3} + \frac{1}{5} - \left(-3 + \frac{4}{3} - \frac{1}{5}\right) \right]$$

$$= \pi \left[6 - \frac{8}{3} + \frac{2}{5} \right]$$

$$= \frac{\pi(90 - 40 + 6)}{15}$$

$$= \frac{56\pi}{15}$$

c)



$R = 1+\sqrt{y}$

$r = 1-\sqrt{y}$

$$V = \int_0^1 \pi\left[(1+\sqrt{y})^2 - (1-\sqrt{y})^2\right] dy$$

$$= \int_0^1 \pi\left[1+2\sqrt{y}+y-(1-2\sqrt{y}+y)\right] dy$$

$$= \int_0^1 4\pi\sqrt{y}\, dy$$

$$= \frac{8\pi}{3} y^{3/2} \Big|_0^1$$

$$= \frac{8\pi}{3}$$