

VOLUMES BY SLICING

Method. To calculate the volume of a solid (using the areas of slices):

- (1) Sketch the solid and a typical cross-section
- (2) Find a formula for $A(x)$
- (3) Find the limits of integration

(4) Evaluate the integral $V = \int_a^b A(x)dx$

1. Find the volume of a solid whose base is the triangle with vertices $(0,0)$, $(0,1)$ and $(2,0)$ and with cross-sections perpendicular to the x -axis are...

- a) squares
- b) semi-circles

Challenge. A water tank has the shape of a sphere with a diameter of 10ft. If the depth of the water in the tank is 7ft, what percent of the tank is empty?

Method (Washer method). To calculate the volume of a solid of rotation, use cross-sections perpendicular to the axis of rotation and $A(x) = \pi [(R(x))^2 - (r(x))^2]$ where $R(x)$ is the outer radius and $r(x)$ is the inner radius.

2. Consider the region enclosed between the parabola $y = x^2$ and the line $y = 1$. Calculate the volume of the solid formed by rotating the region around ...

- a) ... the line $y = 1$
- b) ... the line $y = 2$
- c) ... the line $x = 1$