WS-04 solutions 9/13/20

1. $y = \frac{x^2}{2} - \frac{\ln x}{4}, \quad 1 \le x \le 3$

$$\frac{dy}{dx} = x - \frac{1}{4x}$$

$$1 + \left(\frac{dy}{dx}\right)^2 = 1 + \left(x - \frac{1}{4x}\right)^2 = 1 + x^2 - \frac{1}{2} + \frac{1}{16x^2} = x^2 + \frac{1}{2} + \frac{1}{16x^2}$$

$$= \left(x + \frac{1}{4x}\right)^2$$

$$\sqrt{1 + \left(\frac{dy}{dx}\right)^2} = \sqrt{\left(x + \frac{1}{4x}\right)^2} = x + \frac{1}{4x}$$

$$L = \int_1^3 x + \frac{1}{4x}\, dx = \frac{1}{2}x^2 + \frac{\ln|x|}{4} \Big|_1^3$$

$$= \frac{1}{2}(9) + \frac{\ln 3}{4} - \left(\frac{1}{2} + 0\right) = 4 + \frac{1}{4}\ln 3$$

2. $x = y^{3/2}, \quad 0 \le y \le 4$

$$\frac{dx}{dy} = \frac{3}{2}y^{1/2} \quad \text{so} \quad \sqrt{1 + \left(\frac{dx}{dy}\right)^2} = \sqrt{1 + \frac{9}{4}y}$$

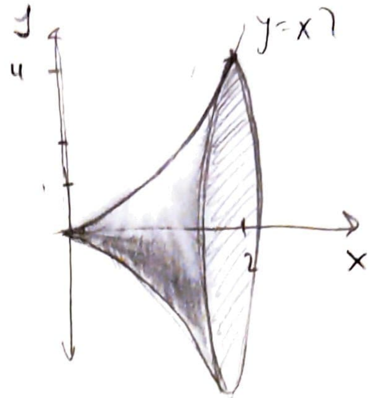
$$L = \int_0^4 \sqrt{1 + \frac{9}{4}y}\, dy \qquad \text{sub } u = 1 + \frac{9}{4}y \quad du = \frac{9}{4}dy$$

$$= \int_1^{10} \sqrt{u}\left(\frac{4}{9}\right) du$$

$$= \frac{2}{3}u^{3/2}\left(\frac{4}{9}\right)\Big|_1^{10}$$

$$= \frac{8}{27}\left[10^{3/2} - 1\right] \approx 9.0734$$

3. $y = x^2, \quad 0 \le x \le 2, \quad$ x-axis



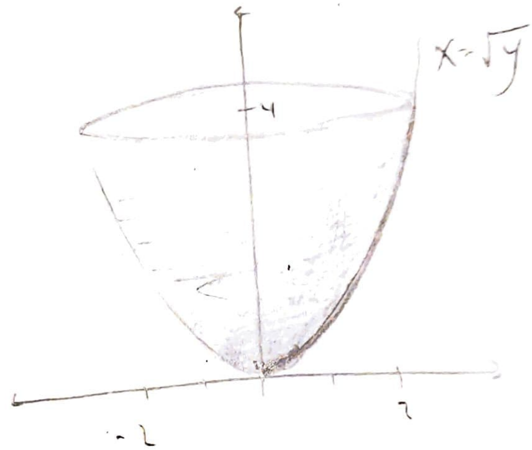
$\frac{dy}{dx} = 2x$

$$SA = \int_0^2 2\pi (x^2) \sqrt{1 + (2x)^2}\, dx$$

$$= \int_0^2 2\pi x^2 \sqrt{1 + 4x^2}\, dx$$

will have to wait for ch. 8 to learn how to evaluate this

4. $x = \sqrt{y}, \quad 0 \le y \le 4, \quad$ y-axis



$\frac{dx}{dy} = \frac{1}{2\sqrt{y}}$

$$SA = \int_0^4 2\pi (\sqrt{y}) \sqrt{1 + \left(\frac{1}{2\sqrt{y}}\right)^2}\, dy$$

$$= \int_0^4 2\pi \sqrt{y\left(1 + \frac{1}{4y}\right)}\, dy$$

$$= \int_0^4 2\pi \sqrt{y + \frac{1}{4}}\, dy \qquad \text{sub } u = y + \frac{1}{4} \quad du = dy$$

$$= \int_{\frac{1}{4}}^{\frac{17}{4}} 2\pi \sqrt{u}\, du$$

$$= 2\pi\left(\frac{2}{3}\right) u^{3/2} \Big|_{1/4}^{17/4}$$

$$= \frac{4\pi}{3}\left[\left(\frac{17}{4}\right)^{3/2} - \left(\frac{1}{4}\right)^{3/2}\right]$$

$$= \frac{\pi}{6}\left[17^{3/2} - 1\right]$$