ARC LENGTHS AND SURFACE AREAS OF SOLIDS OF REVOLUTION

Formula 1. The arc length of $y = f(x)$ over the interval $a \leq x \leq b$ can be calculated using the formula

$$\boxed{L = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx}$$

1. Calculate the arc length of $y = \frac{x^2}{2} - \frac{\ln x}{4}$ for $1 \leq x \leq 3$.

Formula 2. If we are given a curve as $x = g(y)$, then the same combination of geometry and the MVT leads us to another formula for the arc length of the curve over the vertical interval $c \leq y \leq d$:

$$\boxed{L = \int_c^d \sqrt{1 + [g'(y)]^2}\, dy}$$

2. Calculate the arc length of $x = y^{3/2}$ for $0 \leq y \leq 4$.

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Watch this video introducing surfaces of revolution: `https://youtu.be/zUzan1Ma9nE` (click the link to open the video). The teacher is Dr. Trefor Bazett of the University of Cincinnati, who does a very nice job of explaining the ideas (with the help of some nice computer graphics) in only 8.5 minutes.

Formula 3. The surface area of of the surface formed by revolving the arc of the curve $y = f(x)$ over $a \leq x \leq b$ around the x-axis is

$$\boxed{SA = \int_a^b 2\pi f(x)\sqrt{1 + [f'(x)]^2}dx}$$

3. Find an integral giving the area of the surface formed by revolving $y = x^2$ over $0 \leq x \leq 2$ around the x-axis.

Formula 4. The surface area of of the surface formed by revolving the arc of the curve $x = g(y)$ over $c \leq y \leq d$ around the y-axis is

$$\boxed{SA = \int_c^d 2\pi g(y)\sqrt{1 + [g'(y)]^2}dy}$$

4. Find the area of the surface formed by revolving $x = \sqrt{y}$ over $0 \leq y \leq 4$ around the y-axis.