258 F20 9/28 WS-05 solutions

1. $\int \sin^2 x\,dx = \int \sin x \sin x\,dx$

IBP $u = \sin x \quad dv = \cos x\,dx$

$du = \sin x \quad v = -\cos x$

$$= -\sin x \cos x + \int \cos^2 x\,dx$$

$$= -\sin x \cos x + \int 1 - \sin^2 x\,dx$$

$$= -\sin x \cos x + x - \int \sin^2 x\,dx$$

therefore $\int \sin^2 x\,dx = \frac{1}{2}(x - \sin x \cos x) + C$

2. $\sin^2 x + \cos^2 x = 1$

$\tan^2 x + 1 = \sec^2 x$ (divide by $\cos^2 x$)

$1 + \cot^2 x = \csc^2 x$ (divide by $\sin^2 x$)

3. a) $\int \sin^2 x \cos^3 x\,dx = \int \sin^2 x \cos^2 x \cos x\,dx$

$u = \sin x$

$du = \cos x\,dx$

$$= \int \sin^2 x (1 - \sin^2 x) \cos x\,dx$$

$$= \int u^2 (1 - u^2)\,du$$

$$= \int u^2 - u^4\,du$$

$$= \frac{u^3}{3} - \frac{u^5}{5} + C = \frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + C$$

3b) $\int \sin^3 x\,dx = \int (1-\cos^2 x)\sin x\,dx$ $\quad u = \cos x$

$du = -\sin x\,dx$

$$= \int (1-u^2)(-1)\,du$$

$$= \frac{u^3}{3} - u + C$$

$$= \frac{\sin^3 x}{3} - \sin x + C$$

4. $\int \sin^2 x\,dx = \int \frac{1}{2}(1-\cos 2x)\,dx$

$$= \frac{1}{2}\left(x - \frac{1}{2}\sin 2x\right) + C$$

Double angle formula: $\sin 2x = 2\sin x \cos x$

$$= \frac{1}{2}(x - \sin x \cos x) + C$$ ✓