IMPROPER INTEGRALS

1. Determine if the integral converges or diverges. Using a comparison theorem may save you from having to evaluate the integral.

a) $\displaystyle\int_0^4 \frac{1}{\sqrt{4-x}}\,dx$

b) $\displaystyle\int_1^2 \frac{1}{x\ln x}\,dx$

c) $\displaystyle\int_2^\infty \frac{4}{\sqrt{x^3-1}}\,dx$

d) $\displaystyle\int_{-\infty}^\infty e^{-x^2}\,dx$

2. Let R be the region under the curve $y = 1/x$ over the interval $[1, \infty)$. Note that part c of problem 1 showed that R does not have finite area.

a) Find the volume of the solid formed by revolving R around the x-axis. Using the disk method $V = \int_1^\infty \pi\,[f(x)]^2\;dx$.

b) Set up an integral for the surface area of the same solid and determine if it converges or diverges. Recall $SA = \int_1^\infty 2\pi f(x)\sqrt{1+[f'(x)]^2}\;dx$. Do not evaluate the integral; use a comparison to determine if it converges or diverges.

Challenge. The gamma function is defined by $\Gamma(x) = \int_0^\infty t^{x-1}e^{-t}\;dt$.

a) Find $\Gamma(1)$

b) Use IBP to find $\Gamma(2)$

c) Prove that for positive x, $\Gamma(x+1) = x\Gamma(x)$

One consequence of the last fact is that for any natural number n, $\Gamma(n+1) = n!$ (where $n!$ is the factorial: $n! = n(n-1)(n-2)\ldots(3)(2)1$). Hence $\Gamma(x)$ can be thought of as a continuous version of the factorial.

Date: October 8, 2020.