SERIES II

Theorem (Integral estimation theorem)**.** *When the integral test applies to* $\sum a_n$*,*

$$s_k + \int_{k+1}^{\infty} f(x)\,dx \leq \sum_{n=1}^{\infty} a_n \leq s_k + \int_{k}^{\infty} f(x)\,dx$$

1 (Completion)**.** Our goal is to find an estimate for the sum of the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ that is correct to 2 decimal places.

a) Use the integral estimation theorem and s_2 to find an interval containing the sum of the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$. How wide is the interval?
b) Use the integral estimation theorem and s_k to find an interval containing $\sum_{n=1}^{\infty} \frac{1}{n^2}$. This time your interval will depend on k. How wide is the interval (this will also depend on k)?
c) What k makes the width of the interval less that 0.01?
d) Using the value you just found for k, s_k is then our estimate of $\sum_{n=1}^{\infty} \frac{1}{n^2}$ accurate to 2 decimal places.

2 (Completion)**.** Determine if the following series converge or diverge.

a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^3}{n^3+n+1}$
b) $\displaystyle\sum_{n=1}^{\infty} \frac{1+2^n}{3^n}$
c) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)}$

3 (Graded)**.** Use a comparison to show that the following series converge.

a) $\displaystyle\sum_{n=1}^{\infty} \frac{1+2^n}{2+3^n}$
b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n+1}}$

4 (Graded)**.** Use a comparison to show that the following series diverge.

a) $\displaystyle\sum_{n=1}^{\infty} \frac{1+3^n}{2^n-1}$
b) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}(\ln n)}$

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