258 F20 solutions to WS-10 Power Series 11/1

1. $1-3x+9x^2-27x^3+81x^4-\cdots = \sum_{n=0}^{\infty}(-3x)^n$

$= \dfrac{1}{1-(-3x)}$ if $|-3x|<1$

So the radius of convergence comes from isolating $|x|$ in the inequality: $|-3x|<1 \Rightarrow |x|<\frac{1}{3}=R$.

2. $3x+6x^3+12x^5+24x^7+48x^9+\cdots = 3x(1+2x^2+4x^4+8x^6+\cdots)$

$= 3x\sum_{n=0}^{\infty}(2x^2)^n$

$= 3x\left(\dfrac{1}{1-2x^2}\right)$ if $|2x^2|<1$

Find the radius of convergence by isolating $|x|$:

$|2x^2|<1 \Rightarrow |x^2|<\frac{1}{2} \Rightarrow |x|<\frac{1}{\sqrt{2}}=R$

3. $\sum_{n=0}^{\infty}[2(x-5)]^n = \dfrac{1}{1-2(x-5)} = \dfrac{1}{11-2x}$ if $|2(x-5)|<1$

In this case the power series is centered at 5 and the radius of convergence comes from isolating $|x-5|$: $|2(x-5)|<1 \Rightarrow |x-5|<\frac{1}{2}=R$

4. $\sum_{n=1}^{\infty} \frac{(x+2)^n}{n}$ converges if $|x+2| < 1$ and diverges if $|x+2| > 1$ (by direct comparison).

The power series is centered at -2 with radius of convergence 1. To find the interval of convergence we check the values $x=-1$ and $x=-3$:

$x=-1$: $\sum_{n=1}^{\infty} \frac{(-1+2)^n}{n} = \sum_{n=1}^{\infty} \frac{1}{n}$ diverges

$x=-3$: $\sum_{n=1}^{\infty} \frac{(-3+2)^n}{n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ converges

Interval of convergence: $[-3, -1)$

5. $\lim_{n\to\infty} \left| \left(\frac{x-7}{2}\right)^{2n} \right|^{\frac{1}{n}} = \lim_{n\to\infty} \frac{|x-7|^2}{4} = \frac{|x-7|^2}{4} < 1$

Isolate $|x-7|$: $\frac{|x-7|^2}{4} < 1 \Rightarrow |x-7|^2 < 4$

$\Rightarrow |x-7| < 2 = R$

6. $\lim_{n\to\infty} \left| \frac{(x-2)^{n+1}}{(n+1)!} \frac{n!}{(x-2)^n} \right| = \lim_{n\to\infty} \frac{|x-2|}{n+1} = 0$ for all x.

The radius of convergence is ∞.