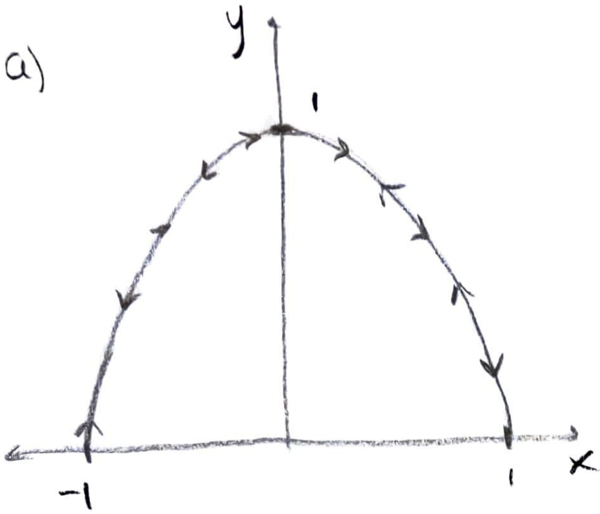
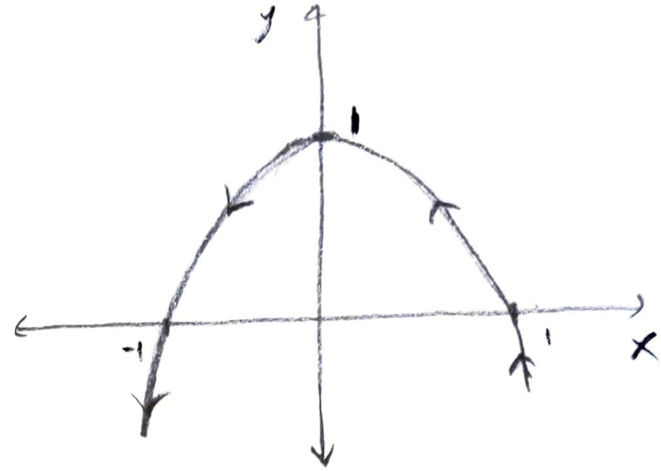
256 F20 Solutions to WS-12 Parametric Equations 11/18

1. a)





b) $x = \cos t$

$y = \sin^2 t = 1 - \cos^2 t = 1 - x^2$

$y = 1 - x^2$

$x = 1 - s \Rightarrow s = 1 - x$

$y = 2s - s^2$

$= 2(1-x) - (1-x)^2$

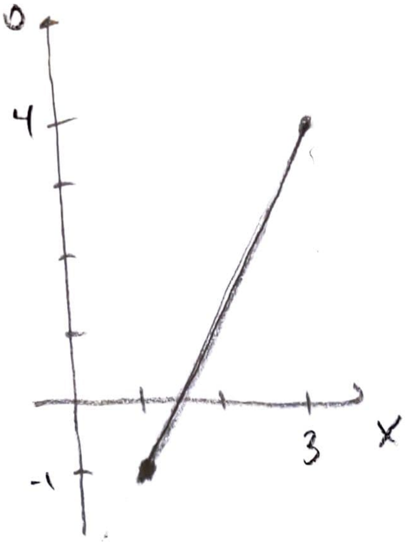
$= 2 - 2x - 1 + 2x - x^2$

$= 1 - x^2$

$y = 1 - x^2$

c) Same curve, but very different ways of tracing out different parts of the curve. The first bounces back and forth across just the piece shown above. The second traces the whole parabola.

2. a)



b) $x = 1 + 2t$

$y = -1 + 5t$

c) Extends the line beyond the segment.

d) The line: $y + 1 = \frac{5}{2}(x-1)$

$y = \frac{5}{2}(x-1) - 1$

$x = t^2$

$y = \frac{5}{2}(t^2 - 1) - 1$

e) slope $\frac{5}{2}$.

In part b: $\frac{dx}{dt} = 2$ and $\frac{dy}{dt} = 5$

In part d: $\frac{dx}{dt} = 2t$ and $\frac{dy}{dt} = 5t$

In both cases $\frac{5}{2} = \frac{dy/dt}{dx/dt}$

3. $x = \cos t$ $\quad dx/dt = -\sin t$ $\quad \frac{dy/dt}{dx/dt} = -2\cos t = -2x$ ✓

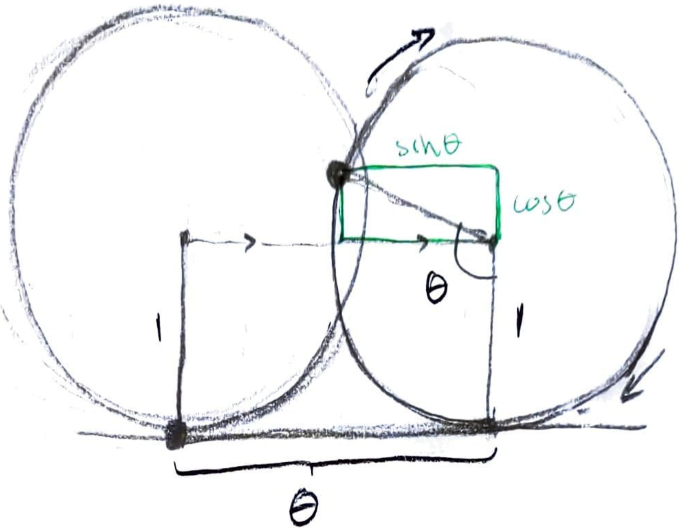
$y = \sin^2 t$ $\quad dy/dt = 2\sin t \cos t$

$x = 1 - s$ $\quad dx/ds = -1$ $\quad \frac{dy/ds}{dx/ds} = 2s - 2 = -2(1-s) = -2x$ ✓

$y = 2s - s^2$ $\quad dy/ds = 2 - 2s$

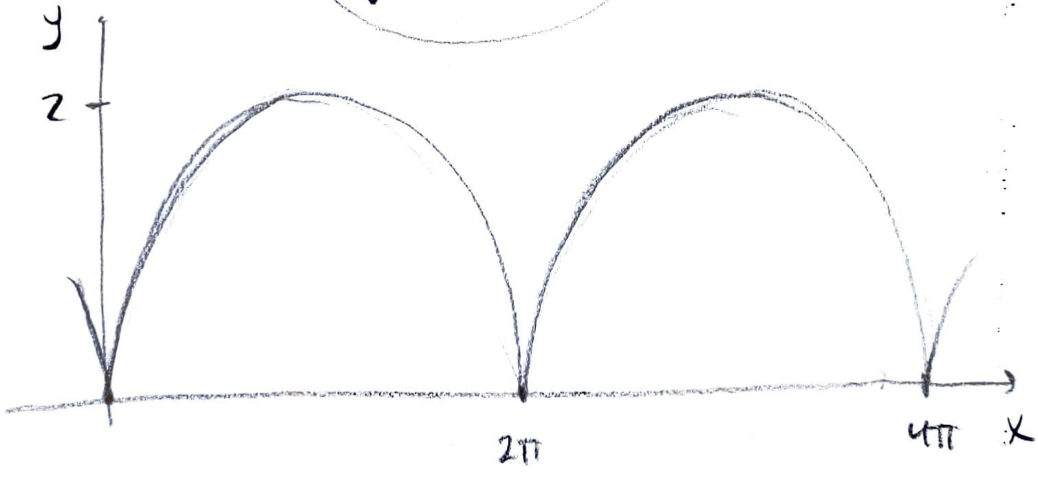
Comparing in both cases, with $\frac{dy}{dx} = \frac{d}{dx}(1 - x^2) = -2x$

4.

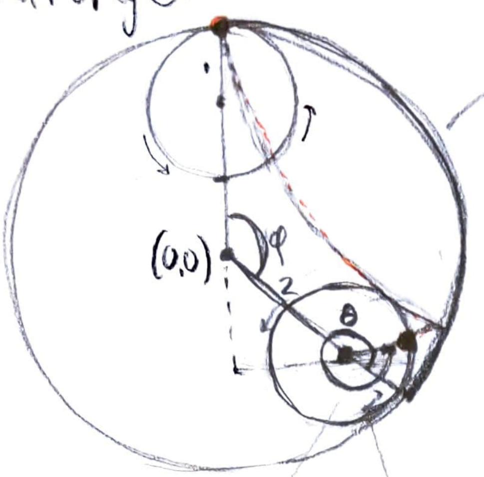


After rolling through θ radians the center is at the point $(\theta, 1)$. The point on the circle's edge is at

$$x = \theta - \sin\theta$$
$$y = 1 - \cos\theta$$



Challenge:



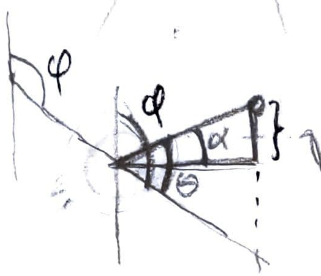
$3\varphi = \theta$ = distance rolled.

Center of the small circle:

$$(2\cos\varphi,\ 2\sin\varphi)$$

Position of the point:

$$x = 2\cos\varphi + \cos\left(\tfrac{\pi}{2} + 2\varphi\right)$$
$$y = 2\sin\varphi + \sin\left(\tfrac{\pi}{2} + 2\varphi\right)$$



$$\alpha = \theta - \left(\varphi - \tfrac{\pi}{2}\right) = \tfrac{\pi}{2} + \theta - \varphi$$
$$= \tfrac{\pi}{2} + 2\varphi$$

shift from center $\sin\alpha$

shift from center $\cos\alpha$