PARAMETRIC EQUATIONS

1. This problem concerns two sets of parametric equations: $(x, y) = (\cos t, \sin^2 t)$ and $(x, y) = (1-s, 2s-s^2)$, both defined for all values of the parameters t and s.

a) Plot both parametric curves (make a table; or do part b first if you're having trouble; or use Desmos).

b) Eliminate the parameters to find Cartesian equations for the curves.

c) What is the difference between the paths traced out by the two pairs of parametric equations?

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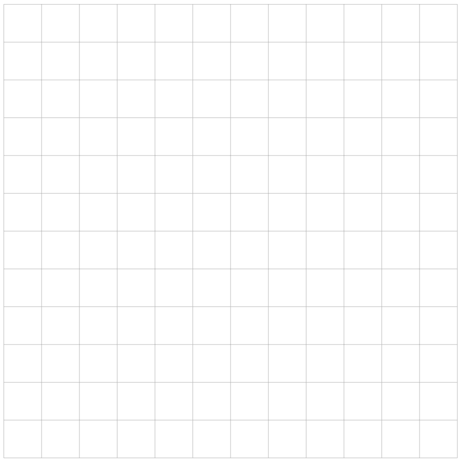
2. The parametric equations $x(t) = x_0 + t(x_1 - x_0)$, $y(t) = y_0 + t(y_1 - y_0)$ with $0 \leq t \leq 1$ describe a line starting at the point (x_0, y_0) and ending at the point (x_1, y_1).

a) Sketch a graph of the line line segment from $(1, -1)$ to $(3, 4)$.



b) Find a parametric description of the line from $(1, -1)$ to $(3, 4)$.

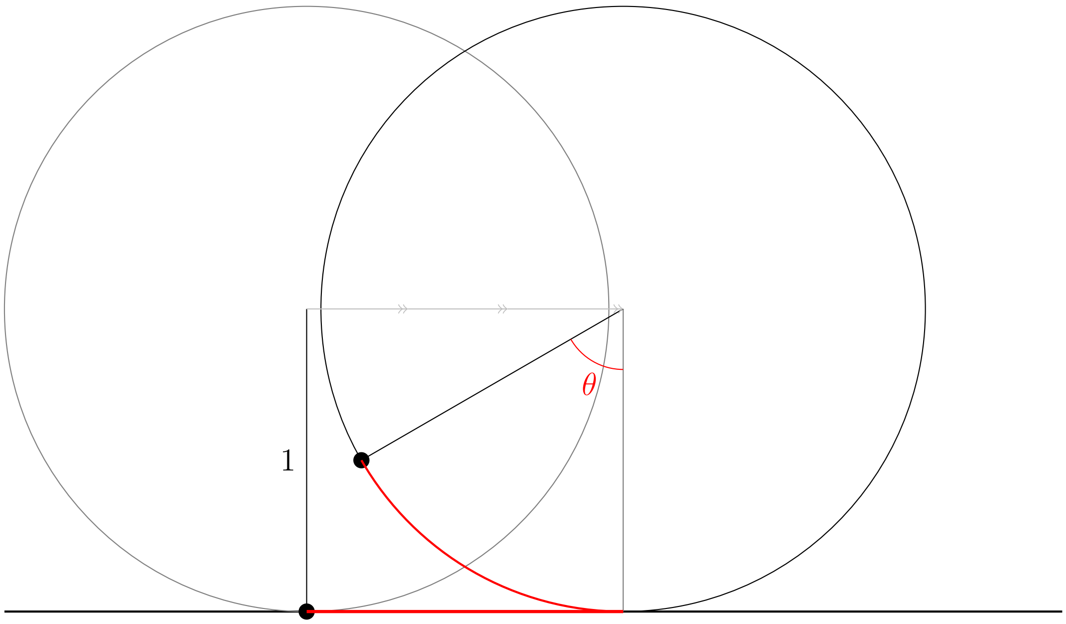
c) Your parametric equations should work just fine if t is less than 0 or greater than 1. What happens for these values of t?

d) Find a parametric description of the same line using $x(t) = t^2$ (don't worry about getting just the segment between the two points).

e) What is the slope of your line (that is, find $\frac{dy}{dx}$)? How is the slope connected to the derivatives $\frac{dx}{dt}$ and $\frac{dy}{dt}$? Check that it works for both descriptions (in parts b and d).

3. Use your formula for $\frac{dy}{dx}$ in terms of $\frac{dx}{dt}$ and $\frac{dy}{dt}$ to find $\frac{dy}{dx}$ for both sets of parametric equations in problem 1. Do your answers agree? Should they agree?

4. Find a parametric description of the curve traced out by a point on the outside of a wheel of radius 1 as the wheel rolls (this curve is called a cycloid). Hint: start with the point on the bottom of the wheel and then use the angle of rotation as the parameter; first locate the center of the wheel then work back to the point.



Challenge. Repeat the previous problem but follow a point on a circle of radius 1 rolling inside a circle of radius 3 (like a spirograph). This produces a hypocycloid