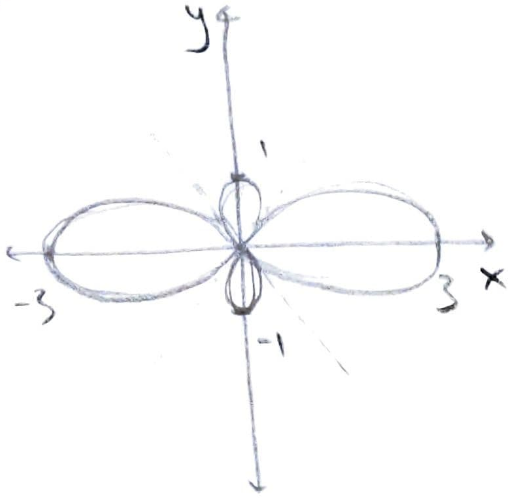
$r = 1 + 2\cos 2\theta$



$\frac{dy}{dx} = ?$

$x = r\cos\theta = (1 + 2\cos 2\theta)\cos\theta$

$\frac{dx}{d\theta} = -4\sin 2\theta \cos\theta - (1+2\cos 2\theta)\sin\theta$

$y = r\sin\theta = (1+2\cos 2\theta)\sin\theta$

$\frac{dy}{d\theta} = -4\sin 2\theta \sin\theta + (1+2\cos 2\theta)\cos\theta$

$$\frac{dy}{dx} = \frac{-4\sin 2\theta \sin\theta + (1+2\cos 2\theta)\cos\theta}{-4\sin 2\theta\cos\theta - (1+2\cos 2\theta)\sin\theta}$$

@ $\theta = \frac{\pi}{3}$: $$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{3}} = \frac{-4\sin\frac{2\pi}{3}\sin\frac{\pi}{3} + \left(1+2\cos\frac{2\pi}{3}\right)\cos\frac{\pi}{3}}{-4\sin\frac{2\pi}{3}\cos\frac{\pi}{3} - \left(1+2\cos\frac{2\pi}{3}\right)\sin\frac{\pi}{3}}$$

$$= \frac{-4\left(\frac{3}{4}\right) + \left(1+(-1)\right)\frac{1}{2}}{-4\left(\frac{\sqrt{3}}{4}\right) - \left(1+(-1)\right)\frac{\sqrt{3}}{2}}$$

$= \sqrt{3}$ (slope of line $\theta = \frac{\pi}{3}$)

where horizontal? $\frac{dy}{dx} = 0$ solve (really $\frac{dy}{d\theta} = 0$)

where vertical? $\frac{dx}{d\theta} = 0$ solve

$1+2\cos 2\theta$

Area inside loops.

Big loop: $A = 2\int_0^{\frac{\pi}{3}} \frac{1}{2}(1+2\cos 2\theta)^2 \, d\theta$

$= \int_0^{\frac{\pi}{3}} 1 + 4\cos 2\theta + 4\cos^2 2\theta \, d\theta$

$= \int_0^{\frac{\pi}{3}} 1 + 4\cos 2\theta + 2 + 2\cos 4\theta \, d\theta$

3

$= 3\theta + 2\sin 2\theta + \frac{1}{2}\sin 4\theta \Big|_0^{\frac{\pi}{3}}$

$= \pi + \sqrt{3} - \frac{\sqrt{3}}{4} = \pi + \frac{3\sqrt{3}}{4}$

Little loop: $A = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 3 + 4\cos 2\theta + 2\cos 4\theta \, d\theta$

Guess + or −

$= 3\theta + 2\sin 2\theta + \frac{1}{2}\sin 4\theta \Big|_{\frac{\pi}{3}}^{\frac{\pi}{2}}$

$= \frac{3\pi}{2} - \left(\pi + \frac{3\sqrt{3}}{4}\right)$

$= \frac{\pi}{2} - \frac{3\sqrt{3}}{4} \quad > 0$

why +? or why not −?

$= 1+2\cos 2\theta$

Arc length of loops.

$\frac{dr}{d\theta} = -4\sin 2\theta$

Big loop: $L = 2\int_0^{\frac{\pi}{3}} \sqrt{(1+2\cos 2\theta)^2 + (-4\sin 2\theta)^2}\, d\theta$

$= 2\int_0^{\frac{\pi}{3}} \sqrt{1+4\cos 2\theta + 4\cos^2 2\theta + 16\sin^2 2\theta}\, d\theta$

$= 2\int_0^{\frac{\pi}{3}} \sqrt{5 + 4\cos 2\theta + 12\sin^2 2\theta}\, d\theta$ ☹

$\approx 2(3.08215)$

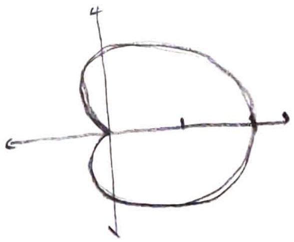
Small loop: $L = 2\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sqrt{(1+2\cos 2\theta)^2 + (-4\sin 2\theta)^2}\, d\theta$

$\approx 2(1.218)$

≈ 2.2436

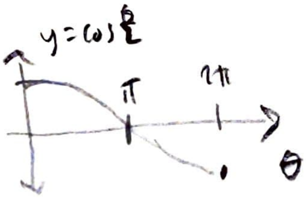
"standard computation time exceeded"

$r = 1+\cos\theta$ $\qquad \frac{dr}{d\theta} = -\sin\theta$



$L = \int_0^{2\pi} \sqrt{(1+\cos\theta)^2 + (-\sin\theta)^2}\, d\theta$

$= \int_0^{2\pi} \sqrt{2+2\cos\theta}\, d\theta$ $\qquad 4\left(\frac{1+\cos\theta}{2}\right) = 4\cos^2\left(\frac{\theta}{2}\right)$



$= \int_0^{2\pi} 2\left|\cos\left(\frac{\theta}{2}\right)\right| d\theta = 2\int_0^{\pi} 2\cos\left(\frac{\theta}{2}\right) d\theta$

$= 8\sin\left(\frac{\theta}{2}\right)\Big|_0^{\pi} = 8(1-0) = \boxed{8}$