POLAR CALCULUS

1. Consider the polar curve $r = 1 + 2\cos(2\theta)$.

a) Convert to parametric using $x = r\cos\theta$ and $y = r\sin\theta$.

b) Use the parametric fomulation to find dy/dx.

c) Calculate $\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{3}}$

Theorem. *The slope of the curve* $r = f(\theta)$ *is*

$$\boxed{\frac{dy}{dx} = \frac{f'(\theta)\sin\theta + f(\theta)\cos\theta}{f'(\theta)\cos\theta - f(\theta)\sin\theta}}$$

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Theorem. *The area of the fan-shaped region between the origin and the curve* $r = f(\theta)$ *for* $\alpha \leq \theta \leq \beta$ *(and* $\beta - \alpha \leq 2\pi$*) is*

$$\boxed{A = \int_{\alpha}^{\beta} \frac{1}{2} r^2 \, d\theta}$$

2. Continue working with the polar curve $r = 1 + 2\cos(2\theta)$. This curve has two big loops and two small loops.

a) Find the area enclosed within one of the big loops by finding the area inside the curve for $0 \leq \theta \leq \frac{\pi}{3}$ and doubling it.

b) Find the area enclosed within one of the small loops.

Theorem. *The length of the polar curve $r = f(\theta)$ for $\alpha \le \theta \le \beta$ (with the curve traced exactly once as θ runs from α to β) is*

$$\boxed{L = \int_{\alpha}^{\beta} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta}$$

3. Find integrals for the lengths of the two sizes of loop in $r = 1 + 2\cos(2\theta)$. These will be (very) hard to evaluate, but you can use a calculator or computer to get approximations.

4. Find the length of the cardioid $r = 1 + \cos\theta$. Hints: you'll need the identity $2\cos^2 u = 1 + \cos(2u)$ and you'll also need to remember that $\sqrt{u^2} = |u|$.