258 F20 12/7 Problems §7.1: 1–51 odd, 61, 69.

1. a) $\frac{d}{dx}(\ln x) = \frac{d}{dx}\left(\int_1^x \frac{1}{t}\,dt\right) = \frac{1}{x}$ by the FTC

b) $\frac{d}{dx}(\ln ax) = \frac{a}{ax} = \frac{1}{x}$

Antidifferentiate both sides: $\ln ax = \ln x + C$.

Let $x=1$: $\ln a = \ln 1 + C$

$= 0 + C$

Therefore $\ln ax = \ln a + \ln x$.

c) $\frac{d}{dx}(\ln x^a) = \frac{ax^{a-1}}{x^a} = \frac{a}{x}$

Antidifferentiate: $\ln(x^a) = a\ln x + C$

Let $x=1$: $0 = C$.

Therefore $\ln(x^a) = a\ln x$.

d) combine parts b and c.

Exponential? Def. e is the number such that

$\ln e = 1$

$\hookrightarrow \int_1^e \frac{1}{x}\,dx$ area interpretation.

2\. a) $\frac{d}{dx}(x = \ln y) \Rightarrow 1 = \frac{y'}{y} \Rightarrow y = y'$

Since $y = e^x$, $\frac{d}{dx}(e^x) = e^x$.

b) $\ln(e^a e^x) = \ln e^a + \ln e^x = a + x$ by property 1b.

Therefore $e^a e^x = e^{a+x}$.

c) $\ln(e^{-a} e^x) = \ln e^{-a} + \ln e^x = -a + x$

Therefore $e^{-a} e^x = e^{-a+x}$

d) $\ln((e^x)^a) = a \ln e^x = ax$.

Therefore $(e^x)^a = e^{ax}$.

3\. $b^{\log_b x} = e^{\ln b \left(\frac{\ln x}{\ln b}\right)} = e^{\ln x} = x$ ✓

$\log_b(b^x) = \frac{\ln(e^{x \ln b})}{\ln b} = \frac{x \ln b}{\ln b} = x$ ✓

4. a) $\Gamma(1) = \int_0^\infty e^{-t}\,dt = \lim_{h\to\infty} -e^{-t}\Big|_0^h = \lim_{h\to\infty} -e^{-h} + 1 = 1$

b) $\Gamma(2) = \int_0^\infty t\,e^{-t}\,dt$ $\quad u = t \quad du = dt$

$\quad dv = e^{-t}\,dt \quad v = -e^{-t}$

$= -te^{-t}\Big|_0^\infty + \int_0^\infty e^{-t}\,dt \to \Gamma(1)$

$= 0 + 1 = 1$

c) $\Gamma(3) = \int_0^\infty t^2 e^{-t}\,dt$ $\quad u = t^2 \quad du = 2t\,dt$

$\quad dv = e^{-t}\,dt \quad v = -e^{-t}$

$= -t^2e^{-t}\Big|_0^\infty + 2\int_0^\infty t\,e^{-t}\,dt \to \Gamma(2)$

$= 0 + 2\Gamma(2) = 2$

d) $\Gamma(n) = (n-1)!$