MATH 259 CHAPTER 10 EXAM FEBRUARY 28, 2014

INSTRUCTIONS: Answer all 11 problems. Show your work: even correct answers may receive little or no credit if a method of solution is not shown. Calculators, notes, cell phones, and other materials are not permitted.

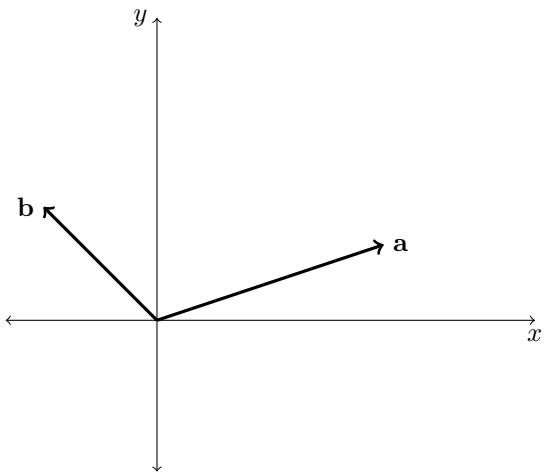
NAME. _____

You may find the following helpful:

- $\text{comp}_{\mathbf{a}}\mathbf{b} = \dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|}$
- $\mathbf{proj}_{\mathbf{a}}\mathbf{b} = \left(\dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|^2}\right)\mathbf{a}$
- $\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$
- $\mathbf{T}(t) = \dfrac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|}$
- $\mathbf{N}(t) = \dfrac{\mathbf{T}'(t)}{|\mathbf{T}'(t)|}$
- $\mathbf{B}(t) = \mathbf{T}(t)\times\mathbf{N}(t)$
- $\kappa = \dfrac{|\mathbf{r}'(t)\times\mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$

1. Find the area of the parallelepiped determined by the vectors $\langle 1,1,0\rangle$, $\langle 2,0,2\rangle$, and $\langle -1,1,1\rangle$.

2. Vectors $\mathbf{a}$ and $\mathbf{b}$ are shown. Draw the vectors $\mathbf{a} + \mathbf{b}$ and $\mathbf{a} - \mathbf{b}$. Label your vectors clearly.



3. Find an equation for the plane containing the line $x = 2t$, $y = 3 - t$, $z = 1 + 3t$ and the point $(1, 3, -2)$.

4. Find the work done by a force $\mathbf{F} = \langle 2, 1, 1 \rangle$ that moves an object from $(0, 0, 0)$ to $(1, 3, 5)$ along a straight line (with measurements in Newtons and meters, respectively).

5. Find the point of intersection of the line $x = 1 + t$, $y = -t$, $z = 1 + 3t$ and the plane $3x + 2y + z = 6$.

6. Determine the distance from the origin to the plane $3x + 2y + z = 4$.

7. This question concerns the quadric surface $\frac{x^2}{4} + y^2 - z = 1$.

a) Is the intersection of the surface with $z = 0$ an ellipse, a parabola, or a hyperbola?

b) Is the intersection of the surface with $y = 3$ an ellipse, a parabola, or a hyperbola?

8. Find an equation for the tangent line to the space curve $\mathbf{r}(t) = \langle t^2, e^t, e^{-t} \rangle$ at the point $(0, 1, 1)$.

9. Find an equation for the curve of intersection of the cylinder $y = z^2$ with the paraboloid $x = 3z^2 + y^2$.

10. Find the length of the curve $\mathbf{r}(t) = \langle \cos(2t), t, \sin(2t) \rangle$ for $0 \leq t \leq 3$.

11. Calculuate the curvature of $\mathbf{r}(t) = \langle \cos t, \sin t, t^2 \rangle$ when $t = \frac{\pi}{2}$.