Ch 11 Exam Review Solutions 4/1/14

1. $\vec{v}(t) = \langle -2\sin t, 3, 2\cos t \rangle$

$\vec{a}(t) = \langle -2\cos t, 0, -2\sin t \rangle$

$v(t) = \sqrt{4\sin^2 t + 9 + 4\cos^2 t} = \sqrt{13}$

2. $\vec{F} = m\vec{a}$

$\vec{v}(t) = 3t^2\vec{i} + 2t\vec{j} + \vec{k}$

$\vec{a}(t) = 6t\vec{i} + 2\vec{j}$

Hence $\vec{F}(t) = 2\vec{a}(t) = 12t\,\vec{i} + 4\vec{j}$

3. $\vec{v}(t) = \langle 2t, 3t^2, 4t^3 \rangle + \vec{c}$

$\langle 1, 0, 0 \rangle = \vec{v}(0) = \langle 0, 0, 0 \rangle + \vec{c}$. Hence $\vec{v}(t) = \langle 2t+1, 3t^2, 4t^3 \rangle$

$\vec{r}(t) = \langle t^2 + t, t^3, t^4 \rangle + \vec{c}$

$\langle 0, 1, -1 \rangle = \vec{r}(0) = \langle 0, 0, 0 \rangle + \vec{c}$. Hence $\vec{r}(t) = \langle t^2 + t, t^3 + 1, t^4 - 1 \rangle$

4. $D_{\vec{u}} f(0,0) \approx 0$

5. $\displaystyle \lim_{(x,y)\to(0,0)} \frac{x^4 - y^4}{x^2 + y^2} = \lim_{(x,y)\to(0,0)} \frac{(x^2-y^2)(x^2+y^2)}{x^2+y^2} = \lim_{(x,y)\to(0,0)} x^2 - y^2 = 0$

6. when $1 + x - y \geq 0$. $\{(x,y) \mid y \leq 1 + x\}$

7. $\displaystyle f_x(x,y) = \frac{(x+y)^2 - 2x(x+y)}{(x+y)^4} = \frac{x+y-2x}{(x+y)^3} = \frac{y-x}{(x+y)^3}$

$\displaystyle f_y(x,y) = \frac{-2x}{(x+y)^3}$

8. $\displaystyle y\frac{\partial z}{\partial x} + \ln y = 2z\frac{\partial z}{\partial x}$ so $\displaystyle \frac{\partial z}{\partial x} = \frac{\ln y}{2z - y}$

$\displaystyle z + y\frac{\partial z}{\partial y} + \frac{x}{y} = 2z\frac{\partial z}{\partial y}$ so $\displaystyle \frac{\partial z}{\partial y} = \frac{\frac{x}{y} + z}{2z - y}$

9. $f_x(x,y) = 2\cos(2x+3y)$ $f_{xy}(x,y) = -6\sin(2x+3y)$ $f_{xyx}(x,y) = -12\cos(2x+3y)$

10. $\frac{\partial z}{\partial x} = \frac{1}{x-2y}$ $\quad \frac{\partial z}{\partial x}\Big|_{(3,1)} = \frac{1}{3-2} = 1$

$\frac{\partial z}{\partial y} = \frac{-2}{x-2y}$ $\quad \frac{\partial z}{\partial y}\Big|_{(3,1)} = \frac{-2}{3-2} = -2$

Tangent plane: $z = x-3 + (-2)(y-1)$

11. $\frac{\partial z}{\partial x} = 2xe^{xy} + x^2ye^{xy}$ $\quad \frac{\partial z}{\partial y} = x^3e^{xy}$

$\frac{\partial x}{\partial t} = 5s-r$ $\quad \frac{\partial y}{\partial t} = \frac{rs}{t}$

$\frac{\partial z}{\partial t} = \left(2xe^{xy} + x^2ye^{xy}\right)(5s-r) + x^3e^{xy}\left(\frac{rs}{t}\right)$

12. $\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}$ when $F(x,y,z) = \cos(x+y+z) - xyz$

$F_x = -\sin(x+y+z) - yz$

$F_z = -\sin(x+y+z) - xy$

$\frac{\partial z}{\partial x} = -\frac{\sin(x+y+z) + yz}{\sin(x+y+z) - xy}$

13. $f_x(x,y) = \frac{-y^2}{x^2}$ $\quad f_y(x,y) = \frac{2y}{x}$

$\nabla f(1,2) = \langle -4, 4\rangle$. f is changing fastest in the direction of $\langle -4, 4\rangle$ (or $\langle -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\rangle$ if a unit vector is required).

14. $f_x(x,y,z) = y^3ze^{xyz}$ $\quad f_y(x,y,z) = 2y\,e^{xyz} + y^2xz\,e^{xyz}$

$f_z(x,y,z) = y^3xe^{xyz}$

$\nabla f(0,1,-1) = \langle -1, 2, 0\rangle$

$D_{\vec{a}} f(0,1,-1) = \langle -1,2,0\rangle \cdot \langle \frac{3}{13}, \frac{4}{13}, \frac{12}{13}\rangle = \frac{-3+8}{13} = \frac{5}{13}$

15. Let $F(x,y,z) = x^2 - z^2 - y$.

$F_x(x,y,z) = 2x$ $\quad F_y(x,y,z) = -1$ $\quad F_z(x,y,z) = -2z$

$\nabla F(4,7,3) = \langle 8, -1, -6\rangle$

Tangent plane: $8(x-4) + (-1)(y-7) + (-6)(z-3) = 0$.

16. $f_x(x,y) = y - \frac{1}{x^2} = 0 \iff y = \frac{1}{x^2}$

$f_y(x,y) = x - \frac{1}{y^2} = 0$

sub $y = \frac{1}{x^2}$ in to 2nd equation:

$$0 = x - \frac{1}{\left(\frac{1}{x^2}\right)^2} = x - x^4 = x(1-x^3)$$

Hence $x=0$, $x=1$ give critical pts.

Note that $x=0$ and $y=0$ make $f(x,y)$ undefined and thus local extremes cannot have $x=0$ or $y=0$.

The only critical point is $(1,1)$.

Use the 2nd derivative test.

$f_{xx}(x,y) = \frac{2}{x^3}$ $\quad f_{xx}(1,1) = 2$

$f_{xy}(x,y) = 1$

$f_{yy}(x,y) = \frac{2}{y^3}$ $\quad f_{yy}(1,1) = 2$

$D = 2(2) - 1^2 = 3 > 0$ and $f_{xx}(1,1) > 0$ so $(1,1)$ is a local min.

17. 1. Find critical points in the region.

$f_x(x,y) = 4 - 2x = 0 \iff x = 2$

$f_y(x,y) = 6 - 2y = 0 \iff y = 3$

$(2,3)$ is a C.P. in the region.

2. Check boundaries.

Corners: $(0,0)$, $(0,5)$, $(4,5)$, $(4,0)$

Edges: $y=0$ $\quad g(x) = 4x - x^2 \quad g'(x) = 4 - 2x = 0 \iff x = 2$

$y=5 \quad g(x) = 4x - x^2 + 5 \quad g'(x) = 4 - 2x = 0 \iff x = 2$

$x=0 \quad h(y) = 6y - y^2 \quad h'(y) = 6 - 2y = 0 \iff y = 3$

$x=4 \quad h(y) = 6y - y^2$ same

Edge CPs: $(2,0)$, $(2,5)$, $(0,3)$, $(4,3)$

3. Compare: $f(0,0) = 0$ $\quad f(0,5) = 5$ $\quad f(4,5) = 16 + 30 - 16 - 25 = 5$

$f(4,0) = 0$ $\quad f(2,0) = 4$ $\quad f(2,5) = 9$ $\quad f(0,3) = 9$ $\quad f(4,3) = 9$

$\boxed{f(2,3) = 13 \text{ max.}}$ $\boxed{\text{Min of } 0 \text{ at } (0,0) \text{ and } (4,0).}$

18. Minimize $d^2 = x^2+y^2+z^2$ given constraint $y^2 = 9 + xz$

$f(x,z) = x^2 + (9+xz) + z^2$

$f_x(x,z) = 2x + z = 0 \iff z = -2x$

$f_z(x,z) = x + 2z = 0$

$x + 2(-2x) = 0$

$x - 4x = -3x = 0 \iff x = 0$

C.P.s $x=0$, $z=0$, $y = \pm 3$.

$(0,3,0)$ and $(0,-3,0)$

Distance to origin at these points: 3.

This must be a minimum, since the surface includes points further from the origin: $(7,4,1)$ for example.