INSTRUCTIONS: Answer all problems. Show your work: even correct answers may receive little or no credit if a method of solution is not shown. Calculators, notes, cell phones, and other materials are not permitted.

NAME. ___________________________________

You may find the following helpful:

- An equation for the tangent plane to the level surface $F(x, y, z) = k$ at the point (x_0, y_0, z_0):

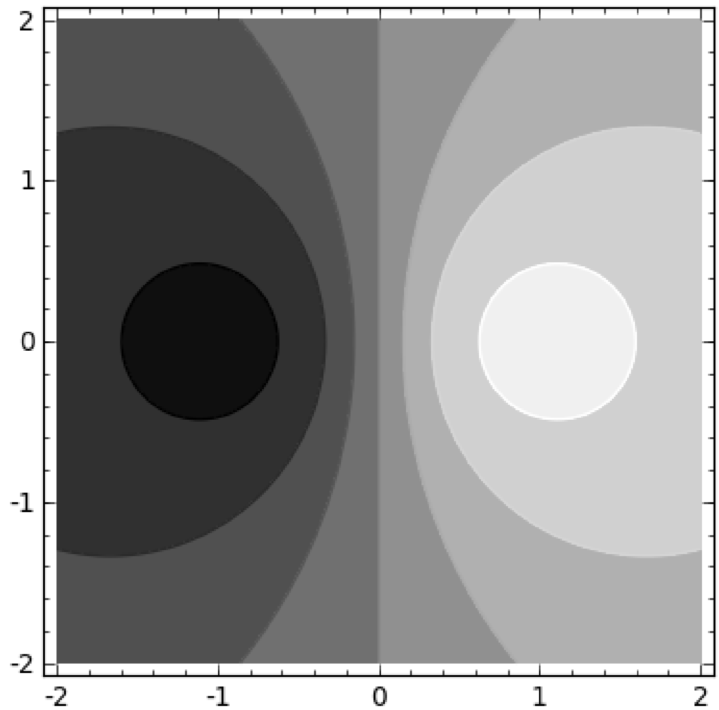
$$F_x(x_0, y_0, z_0)(x - x_0) + F_y(x_0, y_0, z_0)(y - y_0) + F_z(x_0, y_0, z_0)(z - z_0) = 0$$

- Second derivative test for a critical point (a, b) of $f(x, y)$:

$$D(a, b) = f_{xx}(a, b)f_{yy}(a, b) - [f_{xy}(a, b)]^2$$

If $D > 0$ and $f_{xx}(a, b) > 0$, then (a, b) is a local minimum;
If $D > 0$ and $f_{xx}(a, b) < 0$, then (a, b) is a local maximum;
If $D < 0$, then (a, b) is a saddle point.

1. In the contour plot of $f(x, y)$ shown below lighter shades are higher, the x-axis is horizontal, the y-axis is vertical, and the point $(0, 0)$ is in the center.



a) Estimate the value of $D_{\mathbf{u}}f(0, 0)$ when $\mathbf{u} = \langle 0, -1 \rangle$.

b) Is $D_{\mathbf{u}}f(0, 0)$ positive or negative when $\mathbf{u} = \langle 1, 0 \rangle$?

2. Find the position function of a particle with velocity $\mathbf{v}(t) = \langle 0, 6t, 4e^{2t} \rangle$ and initial position $\mathbf{r}(0) = \langle 1, 0, 1 \rangle$.

3. Explain why the limit does not exist: $\lim_{(x,y)\to(0,0)} \dfrac{x^2 - 3y^2}{x^2 + y^2}$.

4. Find an equation for the tangent plane to the surface $z = \sqrt{x - y}$ at the point $(5, 1, 2)$.

5. Find $\frac{\partial w}{\partial t}$ if $w = xe^{yz}$, $x = s \ln t$, $y = s$, and $z = ts$.

6. Find $\frac{\partial z}{\partial x}$ when $xyz = \cos z$.

7. Find the maximum rate of change of the function $f(x, y) = xy^2 - x^2$ at the point $(1, 3)$ and the direction in which it occurs (the direction does not need to be a unit vector).

8. Calculate $D_{\mathbf{u}} f(1, 2)$ for $f(x, y) = \frac{y^2}{x}$ and $\mathbf{u} = \left\langle \frac{2}{3}, \frac{\sqrt{5}}{3} \right\rangle$.

9. Find and equation for the tangent plane to the surface $x = yz - z^2$ at the point $(3, 4, 3)$.

10. Find the critical points of $f(x, y) = x^3 - 6xy + 4y^3$ and determine if each is a local minimum, local maximum, or a saddle point.