## Chapter 12 Exam

INSTRUCTIONS: Answer all problems. Show your work: even correct answers may receive little or no credit if a method of solution is not shown. Calculators, notes, cell phones, and other materials are not permitted.

NAME.

**1.** Evaluate the definite integral  $\iint_R 2xy \ dA$  where R is the region bounded by the lines  $y = 0, \ y = 2x$ , and x = 1.

**2.** Evaluate the integral by first reversing the order of integration:  $\int_0^2 \int_{\frac{y}{2}}^1 e^{x^2} dx dy$ .

**3.** Find an integral expression for the moment  $M_x$  of the lamina with density 1 occupying the region between the parabolas  $y = x^2$  and  $y = 8 - x^2$ . Do not evaluate the integral.

4. Find an integral expression for the volume of the solid under the surface  $z = x^2y$  and above the triangular region in the xy-plane with vertices (0,0), (1,0), and (0,1). Do not evaluate the integral.

5. Find the volume of the solid below the paraboloid  $z = 9 - x^2 - y^2$ , above the xy-plane, and outside the cylinder  $x^2 + y^2 = 1$ .

6. Find an integral expression in spherical coordinates for the moment  $M_{xz}$  of the solid inside the sphere  $x^2 + y^2 + z^2 = 9$ , outside the sphere  $x^2 + y^2 + z^2 = 1$  and above z = 0. Do not evaluate the integral.

7. Let R be the part of a (solid) sphere of radius 1 in the first octant. Express the integral  $\iiint_R z \, dV$  in Cartesian, Cylindrical, and Spherical coordinates.

8. The transformation x = 2u, y = 3v transforms the half disk  $D = \{(u, v) \mid u^2 + v^2 \le 1, v \ge 0\}$  into the half ellipse  $E = \{(x, y) \mid 9x^2 + 4y^2 = 36, y \ge 0\}$ . Use this transformation to evaluate the integral  $\iint_E y \, dA$ . Hint: use another change of variables to polar coordinates.