Math 259

Chapter 9 Exam

January 31, 2014

Instructions: Answer all 11 problems. Show all your work: even correct answers may receive little or no credit if a method of solution is not shown. Calculators, notes, cell phones, and other materials are not permitted.

Name: ____________________________

You may find the following helpful:

- Half-angle formulas: $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$ and $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$;

- Derivative of a polar curve: $\frac{dy}{dx} = \frac{\frac{dr}{d\theta} \sin \theta + r \cos \theta}{\frac{dr}{d\theta} \cos \theta - r \sin \theta}$;

- Arc length of a polar curve: $L = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \, d\theta$;

- Conic sections with foci at the origin and directrices parallel to an axis: $r = \frac{ed}{1 \pm e \cos \theta}$ or $r = \frac{ed}{1 \pm e \sin \theta}$;

- Some values of $\tan \theta$: $\tan 0 = 0$, $\tan(\frac{\pi}{6}) = \frac{1}{\sqrt{3}}$, $\tan(\frac{\pi}{4}) = 1$, $\tan(\frac{\pi}{3}) = \sqrt{3}$, and $\tan(\frac{\pi}{2})$ is undefined.

1. Use the following graphs of $x = f(t)$ and $y = g(t)$ to sketch the parametric curve $x = f(t)$, $y = g(t)$. Indicate the direction in which the curve is traced as $t$ increases and give coordinates for axis intercepts.
2. Eliminate the parameter to find a Cartesian equation for the curve with parametric equations \( x = t^3, \ y = 2t - 1 \).

3. Find the slopes of the two lines tangent to the parametric curve \( x(t) = t - t^{-1}, \ y(t) = 1 + t^2 \) at the point \((x, y) = (0, 2)\).
4. Find \( \frac{d^2y}{dx^2} \) for \( x(t) = \sin t \), \( y(t) = \cos t \).

5. Find the area enclosed between the parametric curve \( x = 1 + t^2 \), \( y = t - t^2 \) and the \( x \)-axis. Hint: the \( x \)-axis is the line \( y = 0 \).
6. Convert the Cartesian coordinates to polar coordinates.
   
a) \((x, y) = (0, 2)\) \((r, \theta) =\) 

b) \((x, y) = (-1, 1)\) \((r, \theta) =\) 

c) \((x, y) = (1, -\sqrt{3})\) \((r, \theta) =\) 

7. Sketch the polar curve \(r = \cos \left(\theta \frac{1}{2}\right)\) for \(0 \leq \theta \leq 2\pi\). Give coordinates for axis intercepts.
8. Find the slope of the line tangent to the polar curve \( r = 2 + \sin(3\theta) \) when \( \theta = \frac{\pi}{3} \).

9. Determine the area of one loop of the polar curve \( r = \sin 3\theta \).
10. Find an integral giving the arc length of the polar curve \( r = \cos \left( \frac{\theta}{4} \right) \). There is no need to evaluate your integral.

11. Determine if the conic section defined by \( r = \frac{6}{4 - 2 \sin \theta} \) is an ellipse, a parabola, or a hyperbola and find a Cartesian equation for its directrix.