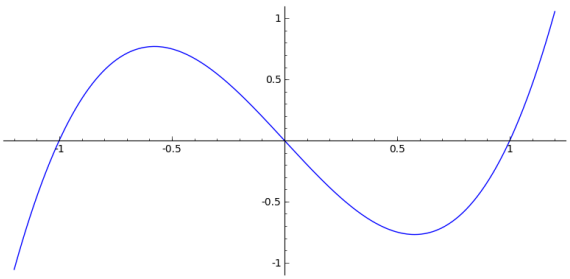
INSTRUCTIONS: Answer all 11 problems. Show all your work: even correct answers may receive little or no credit if a method of solution is not shown. Calculators, notes, cell phones, and other materials are not permitted.

NAME. _____

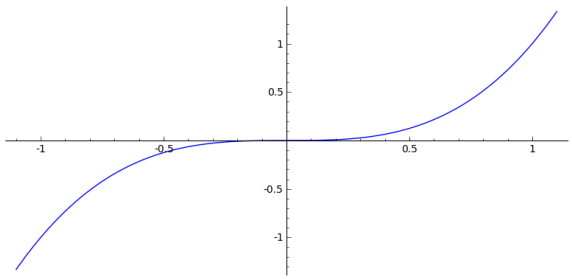
You may find the following helpful:

- Half-angle formulas: $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$ and $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$;
- Derivative of a polar curve: $\dfrac{dy}{dx} = \dfrac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$;
- Arc length of a polar curve: $L = \displaystyle\int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$;
- Conic sections with foci at the origin and directrices parallel to an axis: $r = \dfrac{ed}{1 \pm e\cos\theta}$ or $r = \dfrac{ed}{1 \pm e\sin\theta}$;
- Some values of $\tan\theta$: $\tan 0 = 0$, $\tan(\frac{\pi}{6}) = \frac{1}{\sqrt{3}}$, $\tan(\frac{\pi}{4}) = 1$, $\tan(\frac{\pi}{3}) = \sqrt{3}$, and $\tan(\frac{\pi}{2})$ is undefined.

1. Use the following graphs of $x = f(t)$ and $y = g(t)$ to sketch the parametric curve $x = f(t)$, $y = g(t)$. Indicate the direction in which the curve is traced as t increases and give coordinates for axis intercepts.



$x = f(t)$



$y = g(t)$

2. Eliminate the parameter to find a Cartesian equation for the curve with parametric equations $x = t^3$, $y = 2t - 1$.

3. Find the slopes of the two lines tangent to the parametric curve $x(t) = t - t^{-1}$, $y(t) = 1 + t^2$ at the point $(x, y) = (0, 2)$.

4. Find $\dfrac{d^2y}{dx^2}$ for $x(t) = \sin t$, $y(t) = \cos t$.

5. Find the area enclosed between the parametric curve $x = 1 + t^2$, $y = t - t^2$ and the x-axis. Hint: the x-axis is the line $y = 0$.

6. Convert the Cartesian coordinates to polar coordinates.

a) $(x, y) = (0, 2) \qquad (r, \theta) =$

b) $(x, y) = (-1, 1) \qquad (r, \theta) =$

c) $(x, y) = (1, -\sqrt{3}) \qquad (r, \theta) =$

7. Sketch the polar curve $r = \cos\left(\frac{\theta}{2}\right)$ for $0 \leq \theta \leq 2\pi$. Give coordinates for axis intercepts.

8. Find the slope of the line tangent to the polar curve $r = 2 + \sin(3\theta)$ when $\theta = \frac{\pi}{3}$.

9. Determine the area of one loop of the polar curve $r = \sin 3\theta$.

10. Find an integral giving the arc length of the polar curve $r = \cos\left(\frac{\theta}{2}\right)$. There is no need to evaluate your integral.

11. Determine if the conic section defined by $r = \dfrac{6}{4 - 2\sin\theta}$ is an ellipse, a parabola, or a hyperbola and find a Cartesian equation for its directrix.