INSTRUCTIONS: Answer all problems. Show all your work: even correct answers may receive little or no credit if a method of solution is not shown. Calculators, notes, cell phones, and other materials are not permitted. You may detach this sheet for ease of reference as you work on the exam.

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You may find the following helpful:

- Half-angle formulas: $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$ and $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$;
- Derivative of a polar curve: $\dfrac{dy}{dx} = \dfrac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$;
- Arc length of a polar curve: $L = \displaystyle\int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$;
- Arc length of a curve with vector formula $\mathbf{r}(t)$: $L = \displaystyle\int_a^b |\mathbf{r}'(t)| dt$;
- Conic sections with foci at the origin and directrices parallel to an axis: $r = \dfrac{ed}{1 \pm e\cos\theta}$ or $r = \dfrac{ed}{1 \pm e\sin\theta}$;
- Some values of $\tan\theta$: $\tan 0 = 0$, $\tan(\frac{\pi}{6}) = \frac{1}{\sqrt{3}}$, $\tan(\frac{\pi}{4}) = 1$, $\tan(\frac{\pi}{3}) = \sqrt{3}$, and $\tan(\frac{\pi}{2})$ is undefined.
- $\text{comp}_{\mathbf{a}}\mathbf{b} = \dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|}$
- $\mathbf{proj}_{\mathbf{a}}\mathbf{b} = \left(\dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|^2}\right)\mathbf{a}$
- $\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$
- $\mathbf{T}(t) = \dfrac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|}$
- $\mathbf{N}(t) = \dfrac{\mathbf{T}'(t)}{|\mathbf{T}'(t)|}$
- $\mathbf{B}(t) = \mathbf{T}(t)\times\mathbf{N}(t)$
- $\kappa = \dfrac{|\mathbf{r}'(t)\times\mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$
- An equation for the plane with normal vector $\mathbf{n}$ containing point $\mathbf{r}_0$:

$$\mathbf{n}\cdot(\mathbf{r} - \mathbf{r}_0) = 0$$

- An equation for the tangent plane to the level surface $F(x, y, z) = k$ at the point (x_0, y_0, z_0):

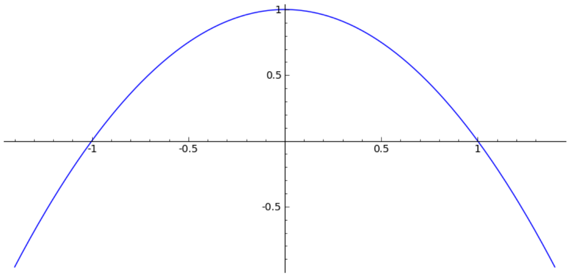
$$F_x(x_0, y_0, z_0)(x - x_0) + F_y(x_0, y_0, z_0)(y - y_0) + F_z(x_0, y_0, z_0)(z - z_0) = 0$$

- Second derivative test for a critical point (a, b) of $f(x, y)$:

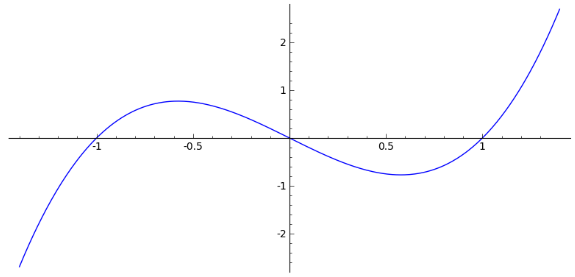
$$D(a, b) = f_{xx}(a, b)f_{yy}(a, b) - [f_{xy}(a, b)]^2$$

If $D > 0$ and $f_{xx}(a, b) > 0$, then (a, b) is a local minimum;
If $D > 0$ and $f_{xx}(a, b) < 0$, then (a, b) is a local maximum;
If $D < 0$, then (a, b) is a saddle point.

1. Use the following graphs of $x = f(t)$ and $y = g(t)$ to sketch the parametric curve $x = f(t)$, $y = g(t)$. Indicate the direction in which the curve is traced as t increases and give coordinates for axis intercepts.



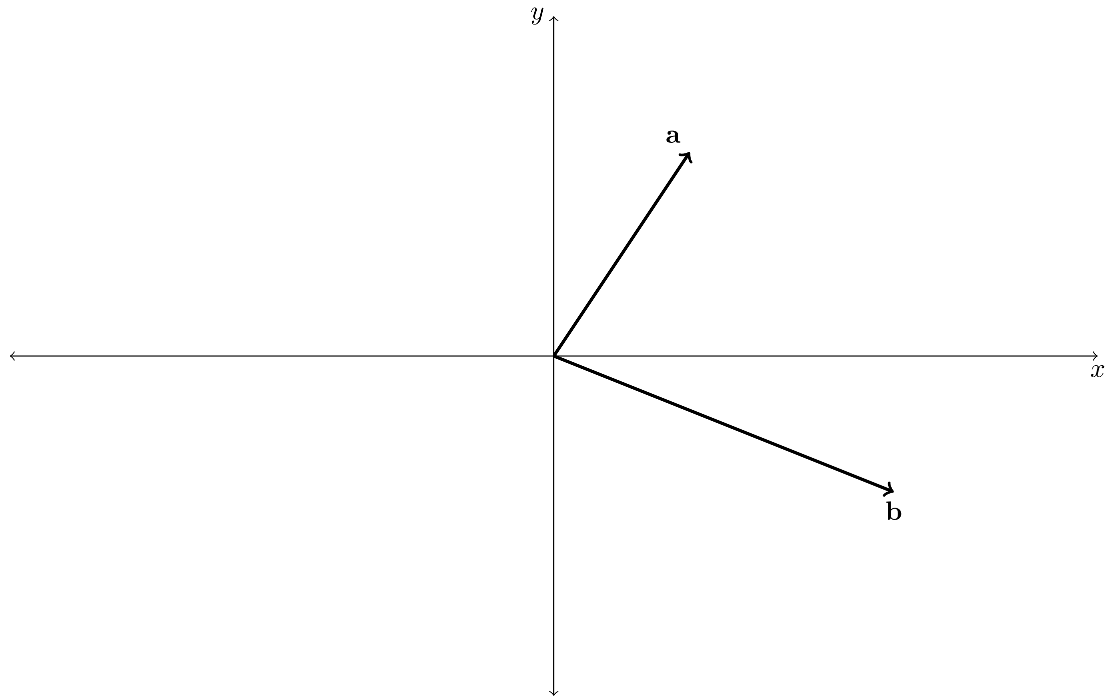
$x = f(t)$



$y = g(t)$

2. Find the slope $\frac{dy}{dx}$ of the line tangent to the parametric curve $x(t) = 1 - t^2$, $y(t) = t^3$ at the point $(x, y) = (0, 1)$.

3. Vectors **a** and **b** are shown. Draw the vectors $\mathbf{a}+\mathbf{b}$ and $\mathbf{a}-\mathbf{b}$ on the same axes. Label your vectors clearly.



4. Find an equation for the plane containing the points $(1,0,0)$, $(1,2,0)$, and $(0,1,1)$.

5. Find the work done by a force $\mathbf{F} = \langle 1, 2, -3 \rangle$ that moves an object from $(0, 0, 0)$ to $(1, 3, -5)$ along a straight line (with measurements in Newtons and meters, respectively).

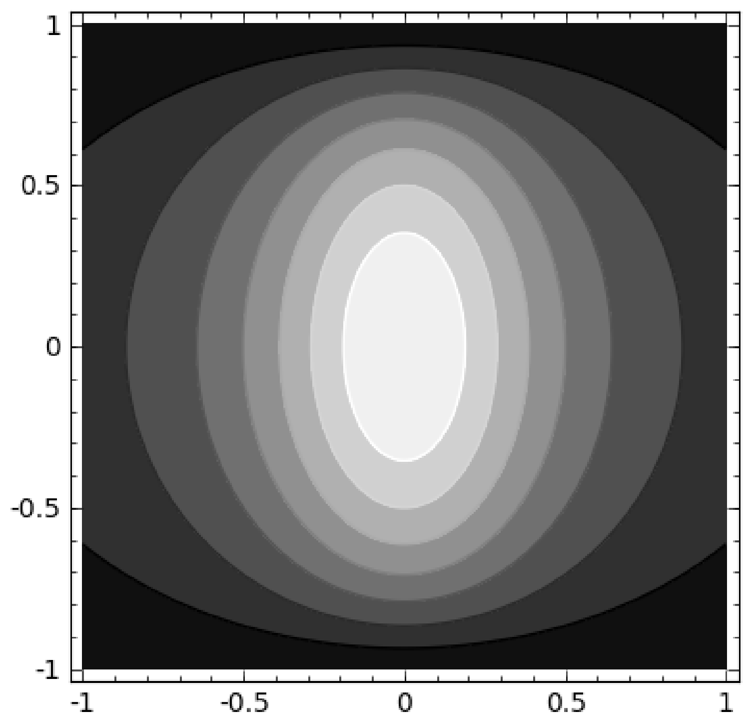
6. Find the point of intersection of the line $x = 1 + 2t$, $y = 1 - t$, $z = 1 + t$ and the plane $x + 2y + z = 6$.

7. Determine the distance from the origin to the plane $x + 2y + z = 6$.

8. Find the length of the curve $\mathbf{r}(t) = \mathbf{i}t + \mathbf{j}\cos(2t) + \mathbf{k}\sin(2t)$ for $0 \le t \le 2$.

9. Find $\frac{\partial z}{\partial y}$ when $e^z = x + x^2 y$.

10. In the contour plot of $z = f(x, y)$ shown below lighter shades are higher, the x-axis is horizontal, the y-axis is vertical, and the point $(0, 0)$ is in the center.



a) Is $D_{\mathbf{u}} f(0, \frac{1}{2})$ positive or negative when $\mathbf{u} = \langle 0, -1 \rangle$?

b) Estimate the gradient vector $\nabla f(0, 0)$.

11. Find an equation for the tangent plane to the surface $z = x^2 + 3xy - y^2$ at the point $(1, 2, 3)$.

12. Find $\frac{\partial z}{\partial t}$ if $z = x^2 + xy + y^2$, $x = s \ln t$, and $y = se^t$.

13. Explain why the limit does not exist: $\lim\limits_{(x,y)\to(0,0)} \dfrac{x+y^2}{x-3y^2}$.

14. Calculate the directional derivative $D_{\mathbf{u}}f(1, \frac{\pi}{2})$ for $f(x, y) = x \sin y$ and $\mathbf{u} = \left\langle \frac{1}{2}, \frac{\sqrt{3}}{2} \right\rangle$.

15. Find the minimum value of $f(x, y, z) = x^2 + y^2 + z^2$ given the constraint $x + 2y + z = 12$. Hint: the method of Lagrange multipliers should work nicely.

16. Evaluate the definite integral $\iint_R 3\sqrt{x^2+y^2}\, dA$ where R is the region inside the circle with polar equation $r = \sin\theta$. Hints: be careful that your limits of integration take you just once around the circle and remember the Pythagorean identity $\cos^2\theta + \sin^2\theta = 1$.

17. Find the mass of the solid with density $d(x,y,z) = \frac{1}{x^2+y^2+z^2}$ occupying the region inside the sphere $x^2+y^2+z^2 = 4$ and outside the sphere $x^2+y^2+z^2 = 1$.

18. Find an integral expression for the moment M_y of the lamina with density $\rho(x,y) = e^x$ occupying the region between the line $x = 0$ and the parabola $x = 4 - y^2$. **Do not evaluate the integral.**

19. Find an integral expression for the volume of the solid under the surface $z = \frac{1}{x+y}$ and above the triangular region in the xy-plane with vertices $(1,0)$, $(0,1)$, and $(1,1)$. **Do not evaluate the integral.**