You may find the following helpful:

- Half-angle formulas: \( \sin^2 x = \frac{1}{2}(1 - \cos 2x) \) and \( \cos^2 x = \frac{1}{2}(1 + \cos 2x) \);

- Derivative of a polar curve: \( \frac{dy}{dx} = \frac{\frac{dr}{d\theta} \sin \theta + r \cos \theta}{\frac{dr}{d\theta} \cos \theta - r \sin \theta} \);

- Arc length of a polar curve: \( L = \int_{a}^{b} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \, d\theta \);

- Arc length of a curve with vector formula \( r(t) \): \( L = \int_{a}^{b} |r'(t)| \, dt \);

- Conic sections with foci at the origin and directrices parallel to an axis: \( r = \frac{ed}{1 \pm e \cos \theta} \) or \( r = \frac{ed}{1 \pm e \sin \theta} \);

- Some values of \( \tan \theta \): \( \tan 0 = 0 \), \( \tan(\frac{\pi}{6}) = \frac{1}{\sqrt{3}} \), \( \tan(\frac{\pi}{4}) = 1 \), \( \tan(\frac{\pi}{3}) = \sqrt{3} \), and \( \tan(\frac{\pi}{2}) \) is undefined.

- \( \text{comp}_a b = \frac{a \cdot b}{|a|} \)

- \( \text{proj}_a b = \left( \frac{a \cdot b}{|a|^2} \right) a \)

- \( a \cdot (b \times c) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \)

- \( T(t) = \frac{r'(t)}{|r'(t)|} \)

- \( N(t) = \frac{T'(t)}{|T'(t)|} \)

- \( B(t) = T(t) \times N(t) \)

- \( \kappa = \frac{|r'(t) \times r''(t)|}{|r'(t)|^3} \)

- An equation for the plane with normal vector \( \mathbf{n} \) containing point \( \mathbf{r}_0 \):

\[
\mathbf{n} \cdot (\mathbf{r} - \mathbf{r}_0) = 0
\]

- An equation for the tangent plane to the level surface \( F(x, y, z) = k \) at the point \((x_0, y_0, z_0)\):

\[
F_x(x_0, y_0, z_0)(x - x_0) + F_y(x_0, y_0, z_0)(y - y_0) + F_z(x_0, y_0, z_0)(z - z_0) = 0
\]

- Second derivative test for a critical point \((a, b)\) of \( f(x, y) \):

\[
D(a, b) = f_{xx}(a, b)f_{yy}(a, b) - [f_{xy}(a, b)]^2
\]

If \( D > 0 \) and \( f_{xx}(a, b) > 0 \), then \((a, b)\) is a local minimum;

If \( D > 0 \) and \( f_{xx}(a, b) < 0 \), then \((a, b)\) is a local maximum;

If \( D < 0 \), then \((a, b)\) is a saddle point.
1. Use the following graphs of \( x = f(t) \) and \( y = g(t) \) to sketch the parametric curve \( x = f(t), y = g(t) \). Indicate the direction in which the curve is traced as \( t \) increases and give coordinates for axis intercepts.

2. Find the slope \( \frac{dy}{dx} \) of the line tangent to the parametric curve \( x(t) = 1 - t^2, y(t) = t^3 \) at the point \((x, y) = (0, 1)\).
3. Vectors $\mathbf{a}$ and $\mathbf{b}$ are shown. Draw the vectors $\mathbf{a} + \mathbf{b}$ and $\mathbf{a} - \mathbf{b}$ on the same axes. Label your vectors clearly.

4. Find an equation for the plane containing the points $(1, 0, 0)$, $(1, 2, 0)$, and $(0, 1, 1)$.
5. Find the work done by a force $\mathbf{F} = (1, 2, -3)$ that moves an object from $(0, 0, 0)$ to $(1, 3, -5)$ along a straight line (with measurements in Newtons and meters, respectively).

6. Find the point of intersection of the line $x = 1 + 2t, y = 1 - t, z = 1 + t$ and the plane $x + 2y + z = 6$. 
7. Determine the distance from the origin to the plane \( x + 2y + z = 6 \).

8. Find the length of the curve \( \mathbf{r}(t) = it + j \cos(2t) + k \sin(2t) \) for \( 0 \leq t \leq 2 \).
9. Find $\frac{\partial z}{\partial y}$ when $e^z = x + x^2y$.

10. In the contour plot of $z = f(x,y)$ shown below lighter shades are higher, the $x$-axis is horizontal, the $y$-axis is vertical, and the point $(0,0)$ is in the center.

a) Is $D_u f(0, \frac{1}{2})$ positive or negative when $u = \langle 0, -1 \rangle$?

b) Estimate the gradient vector $\nabla f(0,0)$. 
11. Find an equation for the tangent plane to the surface \( z = x^2 + 3xy - y^2 \) at the point \((1, 2, 3)\).

12. Find \( \frac{\partial z}{\partial t} \) if \( z = x^2 + xy + y^2, \ x = s \ln t, \) and \( y = se^t \).
13. Explain why the limit does not exist: \( \lim_{(x,y) \to (0,0)} \frac{x + y^2}{x - 3y^2} \).

14. Calculate the directional derivative \( D_uf(1, \frac{\pi}{2}) \) for \( f(x, y) = x \sin y \) and \( u = \left\langle \frac{1}{2}, \frac{\sqrt{3}}{2} \right\rangle \).
15. Find the minimum value of \( f(x, y, z) = x^2 + y^2 + z^2 \) given the constraint \( x + 2y + z = 12 \). Hint: the method of Lagrange multipliers should work nicely.
16. Evaluate the definite integral \( \iint_R 3\sqrt{x^2 + y^2} \, dA \) where \( R \) is the region inside the circle with polar equation \( r = \sin \theta \). Hints: be careful that your limits of integration take you just once around the circle and remember the Pythagorean identity \( \cos^2 \theta + \sin^2 \theta = 1 \).

17. Find the mass of the solid with density \( d(x, y, z) = \frac{1}{x^2 + y^2 + z^2} \) occupying the region inside the sphere \( x^2 + y^2 + z^2 = 4 \) and outside the sphere \( x^2 + y^2 + z^2 = 1 \).
18. Find an integral expression for the moment $M_y$ of the lamina with density $\rho(x, y) = e^x$ occupying the region between the line $x = 0$ and the parabola $x = 4 - y^2$. Do not evaluate the integral.

19. Find an integral expression for the volume of the solid under the surface $z = \frac{1}{x+y}$ and above the triangular region in the $xy$-plane with vertices $(1, 0)$, $(0, 1)$, and $(1, 1)$. Do not evaluate the integral.