Name(s): Solutions

Math 259 All Ellipses January 17, 2014

Definiton. The equation

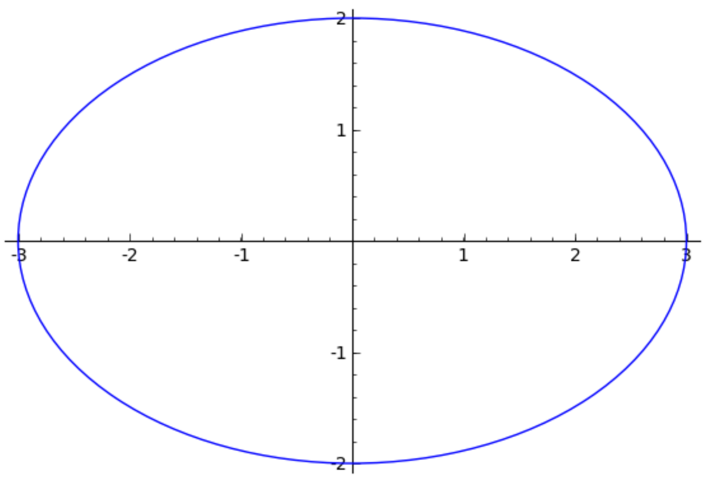
$$\left(\frac{x}{3}\right)^2 + \left(\frac{y}{2}\right)^2 = 1 \tag{1}$$

describes an ellipse centered at $(0,0)$ with a major axis of length 6 and a minor axis of length 4.

1. Show that the parametric equations $\boxed{x = 3\cos\theta}$ and $\boxed{y = 2\sin\theta}$ satisfy equation (1) and hence define the same ellipse.

Solution. $\left(\frac{x}{3}\right)^2 + \left(\frac{y}{2}\right)^2 = \left(\frac{3\cos\theta}{3}\right)^2 + \left(\frac{2\sin\theta}{2}\right)^2 = cos^2\theta + \sin^2\theta = 1$

2. Sketch the ellipse using the parametric equations.



3. Find an equation for the tangent line to the ellipse at the point determined by the given value of θ.

a) $\theta = \frac{\pi}{3}$

b) $\theta = \frac{7\pi}{3}$

Solution. Both values of θ give the same point. The tangent line at that point is $y - \sqrt{3} = -\frac{2}{3\sqrt{3}}\left(x - \frac{3}{2}\right)$

4. Determine the area enclosed by the ellipse. You may need to use the half-angle formula $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$.

Solution. 6π

5. Set up an integral giving the circumference of the ellipse. Do not evaluate this integral.

Solution. $\int_0^{2\pi} \sqrt{9\sin^2\theta + 4\cos^2\theta}\, d\theta$