Name(s):

Math 259 Vectors February 7, 2014

We all know that work is force times distance, at least when you have motion in one dimension. When the force is a vector this still works: If force $\mathbf{F}$ moves an object from point A to point B, then the work done by the force is $W = \mathbf{F} \cdot \mathbf{D}$ where $\mathbf{D} = \overrightarrow{AB}$ is the vector with initial point A and terminal point B (called the *displacement vector*). For the following problems you may want to use theorem 3 of section 10.3: $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos\theta$.

1. Wind blows from the north and exerts a force of 400lb on a sailboat that sails 1000ft southeast. How much work was done by the wind?

2. Suppose the boat in the first problem had sailed 1000ft due east. How much work would you say the wind did in that case? Does this makes sense? Is there something missing that might play a role in the motion of the boat and explain your answer?

3. (Which has nothing to do with force) What is the angle between the diagonal of a cube and one of its edges?

Definiton. If $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, then the *cross product* of $\mathbf{a}$ and $\mathbf{b}$ is

$$\mathbf{a} \times \mathbf{b} = \langle a_2 b_3 - a_3 b_2,\ a_3 b_1 - a_1 b_3,\ a_1 b_2 - a_2 b_1 \rangle.$$

You won't need to use this theorem today, but it's a reason to care about the cross product:

Theorem. *$\boldsymbol{a} \times \boldsymbol{b}$ is orthogonal to both $\boldsymbol{a}$ and $\boldsymbol{b}$.*

4. Let $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ be the standard basis vectors. Calculate $\mathbf{i} \times \mathbf{j}$ and $\mathbf{j} \times \mathbf{i}$. Are they the same?

5. Calculate $\mathbf{i} \times (\mathbf{i} \times \mathbf{j})$ and $(\mathbf{i} \times \mathbf{i}) \times \mathbf{j}$. Are these the same?

Definiton. If a force of $\mathbf{F}$ pushes on a rigid body at a point given by position vector $\mathbf{r}$, then the *torque* τ (relative to the origin) is

$$\tau = \mathbf{r} \times \mathbf{F}$$

6. A 150lb man is hanging from the end of the minute hand of a large clock. The minute hand is 3ft long and it is 10:10. Calculate the torque exerted by the man on the minute hand.