Name(s):
Math 259 | Arc Length and Curvature | February 19, 2014

Definiton. The arc length of the vector function $\mathbf{r}(t) = \langle f(t), g(t), h(t)\rangle$ from $t = a$ to $t = b$ is $\boxed{L = \int_a^b |r'(t)|dt}$.

1. Find the length of the circular helix $\mathbf{r}(t) = \langle \cos t, \sin t, t\rangle$ from $t = 0$ to $t = 2\pi$.

2. Find the *arc length function* $s(t)$ of the circular helix: $s(t) = \displaystyle\int_0^t |\mathbf{r}'(u)|du$.

3. Consider a vector function $\mathbf{r}(t) = \langle f(t), g(t), h(t)\rangle$. Calculate $\frac{ds}{dt}$ for the arc length function $s(t) = \int_0^t |\mathbf{r}'(u)|du$. Hint: the Fundamental Theorem of Calculus might be useful.

Definiton. The *curvature*, κ, of a space curve is, roughly, the rate of change of the direction of the tangent vector (technically it is the magnitude of the rate of change of the unit tangent vector with respect to arc length–we'll discuss this next week) and can be calculated using the formula $\boxed{\kappa = \dfrac{|\mathbf{r}''(t)|}{|\mathbf{r}'(t)|^2}}$.

4. Calculate the curvature of the circular helix $\mathbf{r}(t) = \langle \cos t, \sin t, t \rangle$.

5. Calculate the curvature of a circle of radius a. Do big circles have a higher or lower curvature than small circles?