1. A projectile is fired with angle of elevation α and initial velocity $\mathbf{v}_0$. Our goal is to find $\mathbf{r}(t)$, the position of the object at time t. To simplify matters we assume that air resistance is negligible and that gravity is the only force acting on the projectile.

a) Find the acceleration of the projectile $\mathbf{a}(t)$ (this is a vector).

b) Antidifferentiate $\mathbf{a}(t)$ and use the initial condition $\mathbf{v}(0) = \mathbf{v}_0$ to find the velocity $\mathbf{v}(t)$.

c) Antidifferentiate again to find the position $\mathbf{r}(t)$.

d) Express $\mathbf{v}_0$ as a function of α and initial speed v_0 and substitute this into your formula for $\mathbf{r}(t)$.

2. A projectile is fired from a height of 2 m with an initial velocity of 80 m/s at an angle of 30° above horizontal. Determine how far the projectile travels horizontally before it hits the ground ($y = 0$).

3. It is often convenient to decompose the acceleration of an object into two components: one in the direction of the tangent ($\mathbf{T}$) and one in the direction of the normal ($\mathbf{N}$). Recall that

$$\mathbf{T} = \frac{\mathbf{r}'}{|\mathbf{r}'|} = \frac{\mathbf{v}}{v} \tag{1}$$

and

$$\mathbf{N} = \frac{\mathbf{T}'}{|\mathbf{T}'|} \tag{2}$$

a) Solve equation 1 for $\mathbf{v}$ and differentiate both sides to get a formula for $\mathbf{a}$.

b) Solve equation 2 for $\mathbf{T}'(t)$ and substitute into your answer for part a. You should now have an expression of $\mathbf{a}(t)$ as the sum of scalar multiples of $\mathbf{T}$ and $\mathbf{N}$.

c) It is customary to make the substitution $|\mathbf{T}'| = \kappa v$ where κ is the curvature. Check that this gives you $\mathbf{a} = v'\mathbf{T} + \kappa v^2\mathbf{N}$.

4. The magnitude of the tangential component of acceleration is $a_T = v'$ and the magnitude of the normal component of acceleration is $a_N = \kappa v^2$.

a) Explain (in English) what happens when $a_T = 0$.

b) Explain (in English) what happens when $a_N = 0$.

c) What is the significance of the absence of the binormal vector $\mathbf{B}$ in our decomposition of acceleration?

5. Consider an object with position $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$ (the twisted cubic).

a) Find the tangential and normal components of acceleration (you are welcome to cite old work here).

b) Find the tangential and normal components of acceleration at the points $(0, 0, 0)$ and $(1, 1, 1)$.