Definition. The gradient vector of $f(x, y)$ is

$$\nabla f = f_x(x, y)i + f_y(x, y)j = (f_x(x, y), f_y(x, y))$$

Using the gradient vector, we can write the equation of the tangent plane to the surface $z = f(x, y)$ at $(x_0, y_0, z_0)$ as $z - z_0 = \nabla f(x_0, y_0) \cdot (x - x_0, y - y_0)$.

1. Find an equation for the tangent plane to the surface $z = y + \sin \left( \frac{x}{y} \right)$ at $(0, 3, 3)$.

The tangent plane to $z = f(x, y)$ at $(x_0, y_0, z_0)$ lies close to the surface near $(x_0, y_0, z_0)$. This allows us to use the tangent plane to approximate values of $f(x, y)$ when $(x, y)$ is near $(x_0, y_0)$. For example, the tangent plane to $z = e^x \cos y$ at $(1, 0, e)$ is $z = ex$ and so $e^{0.9} \cos(0.1) \approx 0.9e$ (using a calculator shows that $0.9e \approx 2.446453646$ while $e^{0.9} \cos(0.1) \approx 2.447315341$).

2. Use your tangent plane from the previous problem to approximate $f \left( \frac{1}{5}, \frac{16}{2} \right)$ for $f(x, y) = y + \sin \left( \frac{x}{y} \right)$.

The directional derivative of a function $f$ is the instantaneous rate of change of $f$ in a particular direction. For example, $f_x$ is a directional derivative in the direction of $i$ and $f_y$ is a directional derivative in the direction of $j$. 
**Theorem.** The directional derivative of $f$ in the direction of a unit vector $\mathbf{u}$ is the vector component of $\nabla f$ in the direction of $\mathbf{u}$:

$$D_{\mathbf{u}}f(x,y) = \nabla f \cdot \mathbf{u}$$

3. Calculate $D_{\mathbf{u}}f(0,3)$ for $f(x,y) = y + \sin \left(\frac{z}{y}\right)$ and $\mathbf{u} = \langle \frac{1}{2}, \frac{\sqrt{3}}{2} \rangle$.

4. Determine which direction for $\mathbf{u}$ maximizes the directional derivative $D_{\mathbf{u}}f(x,y)$. Hint: apply the formula $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$ and determine which $\theta$ maximizes the directional derivative.

5. Suppose you are at the point $(0,3,3)$ in the surface $z = y + \sin \left(\frac{z}{y}\right)$. Which direction should you go in order to climb fastest?