NAME(S):

MATH 259 LINEAR APPROXIMATION, DIFFERENTIALS, AND THE GRADIENT VECTOR MARCH 21, 2014

Definiton. The *gradient vector* of $f(x, y)$ is

$$\boxed{\nabla f = f_x(x, y)\mathbf{i} + f_y(x, y)\mathbf{j} = \langle f_x(x, y), f_y(x, y)\rangle}$$

Using the gradient vector, we can write the equation of the tangent plane to the surface $z = f(x, y)$ at (x_0, y_0, z_0) as $z - z_0 = \nabla f(x_0, y_0) \cdot \langle x - x_0, y - y_0\rangle$.

1. Find an equation for the tangent plane to the surface $z = y + \sin\left(\frac{x}{y}\right)$ at $(0, 3, 3)$.

The tangent plane to $z = f(x, y)$ at (x_0, y_0, z_0) lies close to the surface near (x_0, y_0, z_0). This allows us to use the tangent plane to approximate values of $f(x, y)$ when (x, y) is near (x_0, y_0). For example, the tangent plane to $z = e^x \cos y$ at $(1, 0, e)$ is $z = ex$ and so $e^{0.9} \cos(0.1) \approx 0.9e$ (using a calculator shows that $0.9e \approx 2.446453646$ while $e^{0.9} \cos(0.1) \approx 2.447315341$).

2. Use your tangent plane from the previous problem to approximate $f\left(\frac{1}{5}, \frac{16}{5}\right)$ for $f(x, y) = y + \sin\left(\frac{x}{y}\right)$.

The *directional derivative* of a function f is the instantaneous rate of change of f in a particular direction. For example, f_x is a directional derivative in the direction of $\mathbf{i}$ and f_y is a directional derivative in the direction of $\mathbf{j}$.

Theorem. *The directional derivative of f in the direction of a unit vector $\mathbf{u}$ is the vector component of ∇f in the direction of $\mathbf{u}$:*

$$\boxed{D_{\mathbf{u}} f(x, y) = \nabla f \cdot \mathbf{u}}$$

3. Calculate $D_{\mathbf{u}} f(0, 3)$ for $f(x, y) = y + \sin\left(\frac{x}{y}\right)$ and $\mathbf{u} = \langle \frac{1}{2}, \frac{\sqrt{3}}{2} \rangle$.

4. Determine which direction for $\mathbf{u}$ maximizes the directional derivative $D_{\mathbf{u}} f(x, y)$. Hint: apply the formula $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos\theta$ and determine which θ maximizes the directional derivative.

5. Suppose you are at the point $(0, 3, 3)$ in the surface $z = y + \sin\left(\frac{x}{y}\right)$. Which direction should you go in order to climb fastest?