**Proposition.** The mass of a lamina with density \( \rho(x, y) \) occupying region \( D \) in the \( xy \)-plane is

\[
m = \int\int_D \rho(x, y) \, dA.
\]

The moments about the \( x \)-axis and \( y \)-axis of the same lamina are (respectively)

\[
M_x = \int\int_D y \rho(x, y) \, dA \quad \text{and} \quad M_y = \int\int_D x \rho(x, y) \, dA.
\]

The center of mass of the lamina is \((\overline{x}, \overline{y})\) where \( \overline{x} = \frac{M_y}{m} \) and \( \overline{y} = \frac{M_x}{m} \).

1. Find the center of mass of a lamina with density \( \rho(x, y) = y \) occupying the triangular region bounded by \( x = 0, y = x, \) and \( y = 2 - x \).

**Proposition.** The moment of inertia (also called the second moment) of a particle of mass \( m \) at distance \( r \) from the axis of rotation is \( mr^2 \). For a lamina with density \( \rho(x, y) \) occupying region \( D \) in the \( xy \)-plane, the moments of inertia about the \( x \)-axis and \( y \)-axis are (respectively)

\[
I_x = \int\int_D y^2 \rho(x, y) \, dA \quad \text{and} \quad I_y = \int\int_D x^2 \rho(x, y) \, dA.
\]

The moment of inertia about the origin is

\[
I_0 = \int\int_D (x^2 + y^2) \rho(x, y) \, dA.
\]
2. Find the moments of inertia ($I_x$, $I_y$, and $I_0$) of a lamina with density $\rho(x, y) = x$ and occupying $D = \{(x, y) \mid x \geq 0, \ y \geq 0, \ x^2 + y^2 \leq 1\}$.

3. Evaluate the integral $\int\int\int_C xyz \, dV$ where $C = \{(x, y, z) \mid 0 \leq x \leq 1, \ 1 \leq y \leq 2, \ 2 \leq z \leq 3\}$ (this gives the mass of the cube $C$ with density $\rho(x, y, z) = xyz$).