Proposition. The mass of a lamina with density $\rho(x, y)$ occupying region D in the xy-plane is

$$m = \iint_D \rho(x, y)\ dA.$$

The moments about the x-axis and y-axis of the same lamina are (respectively)

$$M_x = \iint_D y\rho(x, y)\ dA \text{ and } M_y = \iint_D x\rho(x, y)\ dA.$$

The center of mass of the lamina is $(\overline{x}, \overline{y})$ where $\overline{x} = \dfrac{M_y}{m}$ and $\overline{y} = \dfrac{M_x}{m}$.

1. Find the center of mass of a lamina with density $\rho(x, y) = y$ occupying the triangular region bounded by $x = 0$, $y = x$, and $y = 2 - x$.

Proposition. The moment of inertia (also called the second moment) of a particle of mass m at distance r from the axis of rotation is mr^2. For a lamina with density $\rho(x, y)$ occupying region D in the xy-plane, the moments of inertia about the x-axis and y-axis are (respectively)

$$I_x = \iint_D y^2\rho(x, y)\ dA \text{ and } I_y = \iint_D x^2\rho(x, y)\ dA.$$

The moment of inertia about the origin is

$$I_0 = \iint_D (x^2 + y^2)\rho(x, y)\ dA.$$

2. Find the moments of inertia (I_x, I_y, and I_0) of a lamina with density $\rho(x,y) = x$ and occupying $D = \{(x,y) \mid x \geq 0,\ y \geq 0,\ x^2 + y^2 \leq 1\}$.

3. Evaluate the integral $\iiint_C xyz\ dV$ where $C = \{(x,y,z) \mid 0 \leq x \leq 1,\ 1 \leq y \leq 2,\ 2 \leq z \leq 3\}$ (this gives the mass of the cube C with density $\rho(x,y,z) = xyz$).