NAME: ____________________________________

INSTRUCTIONS: Answer all 18 problems. Show your work: even correct answers may receive little or no credit if a method of solution is not shown. There is no need to simplify your solutions. Calculators, notes, cell phones, and other materials are not permitted.

Some useful formulas:

- Derivative of a polar curve: $\dfrac{dy}{dx} = \dfrac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$;
- Arc length: $L = \displaystyle\int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$ or $L = \displaystyle\int_\alpha^\beta |\mathbf{r}'(t)|\, dt$
- $\text{comp}_{\mathbf{a}}\mathbf{b} = \dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|}$
- $\mathbf{proj_a b} = \left(\dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|^2}\right)\mathbf{a}$
- $\mathbf{a}\times\mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$
- $\mathbf{T}(t) = \dfrac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|}$
- $\mathbf{N}(t) = \dfrac{\mathbf{T}'(t)}{|\mathbf{T}'(t)|}$
- $\mathbf{B}(t) = \mathbf{T}(t)\times\mathbf{N}(t)$
- $\kappa(t) = \dfrac{|\mathbf{r}'(t)\times\mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$
- An equation for the tangent plane to the level surface $F(x,y,z) = k$ at the point (x_0, y_0, z_0):

$$F_x(x_0,y_0,z_0)(x-x_0) + F_y(x_0,y_0,z_0)(y-y_0) + F_z(x_0,y_0,z_0)(z-z_0) = 0$$

- Distance from the point (x_1, y_1, z_1) to the plane $ax + by + cz + d = 0$: $\dfrac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2+b^2+c^2}}$
- $\mathbf{F} = m\mathbf{a}$.
- $\boldsymbol{\tau} = \mathbf{r}\times\mathbf{F}$.
- Second derivative test for a critical point (a,b) of $f(x,y)$: $D(a,b) = f_{xx}(a,b)f_{yy}(a,b) - [f_{xy}(a,b)]^2$.

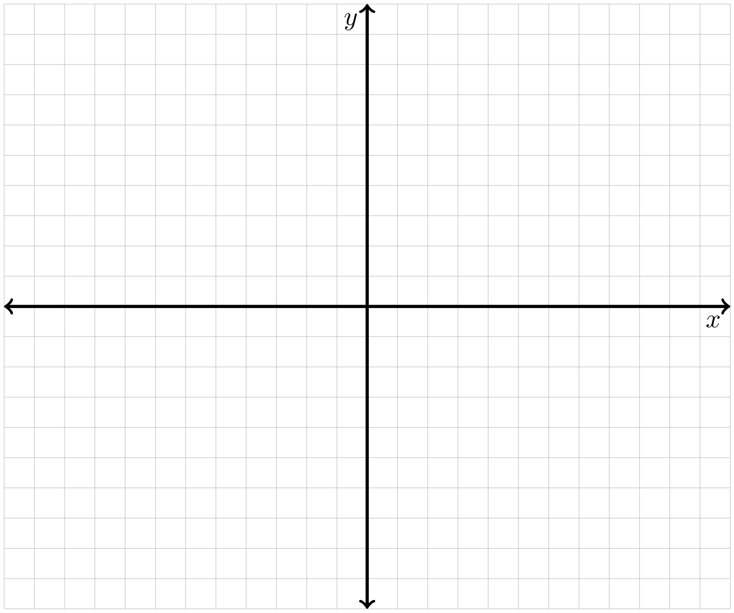
  If $D > 0$ and $f_{xx}(a,b) > 0$, then $f(a,b)$ is a local minimum;

  If $D > 0$ and $f_{xx}(a,b) < 0$, then $f(a,b)$ is a local maximum;

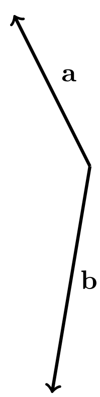
  If $D < 0$, then $f(a,b)$ is not a local extreme;

  If $D = 0$, then the test is inconclusive.
- $dV = dx\, dy\, dz = r\, dz\, dr\, d\theta = \rho^2\sin\phi\, d\rho\, d\theta\, d\phi$.
- Spherical–Cartesian conversions: $x = \rho\sin\phi\cos\theta$, $y = \rho\sin\phi\sin\theta$, $z = \rho\cos\phi$, and $x^2+y^2+z^2 = \rho^2$.
- The Jacobian of the transformation $x = g(u,v)$, $y = h(u,v)$ is $\dfrac{\partial(x,y)}{\partial(u,v)} = \left(\dfrac{\partial x}{\partial u}\right)\left(\dfrac{\partial y}{\partial v}\right) - \left(\dfrac{\partial x}{\partial v}\right)\left(\dfrac{\partial y}{\partial u}\right)$.

1. Sketch the graph of the polar curve $r = 1 + 2\cos\theta$. Label all axis intercepts clearly.



2. Vectors $\mathbf{a}$ and $\mathbf{b}$ are shown. Draw the vectors $\mathbf{a} + \mathbf{b}$ and $\mathbf{a} - \mathbf{b}$ and determine which has greater magnitude. Label your vectors clearly.



3. Determine the slope of the tangent line to the curve $r = 1 - \cos\theta$ at the point $(r, \theta) = \left(1, \frac{\pi}{2}\right)$.

4. Determine the area of the region inside the polar curve $r = \theta$ for $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$.

5. Determine the angle of intersection of the lines $L_1 : \langle x, y, z\rangle = \langle 2t, 2, 2t - 1\rangle$ and $L_2 :\ \langle x, y, z\rangle = \langle s + 1, 1 - s, -2 - s\rangle$.

6. Find an equation for the plane containing the lines $L_1 : x = 6t - 3,\ y = 1 - 3t,\ z = 2t$ and $L_2 : x = 2s - 1,\ y = -s,\ z = 4 - s$.

7. Find a set of parametric equations describing the curve of intersection of the cylinders $z = x^2$ and $x^2 + y^2 = 1$.

8. Calculate the curvature of the space curve $\mathbf{r}(t) = \langle t^2, t^3 - t, t \rangle$ at the point $(1, 0, 1)$.

9. Explain why the limit does not exist: $\lim_{(x,y)\to(0,0)} \dfrac{xy}{(x+y)^2}$.

10. Calculate $\frac{\partial z}{\partial y}$ if $x^2 - y^2 + z^2 - 2z = 4$.

11. Calculate $\frac{\partial w}{\partial t}$ if $w = xe^{yz}$ and $x = r\cos t$, $y = r\sin t$, and $z = r^2 + t^2$.

12. Calculate the directional derivative $D_{\mathbf{u}}f(2,1)$ if $\mathbf{u} = \left\langle \frac{3}{\sqrt{34}}, \frac{5}{\sqrt{34}} \right\rangle$ and $f(x,y) = \dfrac{1}{x^2 - y^2}$.

13. Evaluate the definite integral $\iint_R x\, dA$ where R is the region bounded by the y-axis and the lines $x = y + 2$, and $x = 2y$.

14. Evaluate the iterated integral $\int_0^1 \int_x^1 \cos(y^2)\, dydx$.

15. Calculate the volume of the solid between the paraboloid $z = 4 - x^2 - y^2$ and the xy-plane.

16. Find an integral expression for the moment M_{xy} of the solid with uniform density 1 occupying the spherical region above the cone $\phi = \frac{\pi}{6}$ and inside the sphere $\rho = 3$. Do not evaluate the integral.

17. Find all the critical points of $f(x, y) = xy + \frac{1}{x} + \frac{1}{y}$ and determine if each is a local minimum, local maximum, or neither.

18. Determine the maximum and minimum values of $f(x, y) = 3x + y$ given that $x^2 + y^2 = 10$.