MATH 259 CHAPTER 10 EXAM FEBRUARY 27, 2015

INSTRUCTIONS: Answer all 10 problems. Show your work: even correct answers may receive little or no credit if a method of solution is not shown. Calculators, notes, cell phones, and other materials are not permitted.

NAME. _____

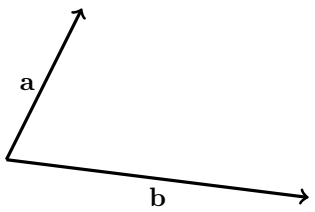
You may find the following helpful:

- $\text{comp}_{\mathbf{a}}\mathbf{b} = \dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|}$
- $\mathbf{proj}_{\mathbf{a}}\mathbf{b} = \left(\dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|^2}\right)\mathbf{a}$
- $\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$
- $\mathbf{T}(t) = \dfrac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|}$
- $\mathbf{N}(t) = \dfrac{\mathbf{T}'(t)}{|\mathbf{T}'(t)|}$
- $\mathbf{B}(t) = \mathbf{T}(t)\times\mathbf{N}(t)$
- $\kappa = \dfrac{|\mathbf{r}'(t)\times\mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$
- Distance from the point (x_1, y_1, z_1) to the plane $ax + by + cz + d = 0$:

$$\frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

- $\mathbf{F} = m\mathbf{a}$.

1. Vectors $\mathbf{a}$ and $\mathbf{b}$ are shown. Draw the vectors $\mathbf{a} + \mathbf{b}$ and $\mathbf{b} - 2\mathbf{a}$. Label your vectors clearly.



2. A wrench 30 cm long lies along the positive y-axis and grips a bolt at the origin. A force of $\langle -1, 1, 4 \rangle$ N is applied to the end of the wrench. Calculate the torque on the bolt.

3. Find an equation for the plane containing the lines $x = 2t$, $y = 3 - t$, $z = 1 + 3t$ and $x = 1 - s$, $y = 3s$, $z = 1$.

4. Find the point of intersection of the line $x = 1 - 3y$, $z = 1$ and the plane $x + 2y + z = 6$.

5. The line $x = 1 + t$, $y = -t$, $z = t$ is parallel to the plane $x + 2y + 2z = 6$. Determine the distance from the line to the plane.

6. Find a set of parametric equations for the curve of intersection of the cylinder $x^2 + z^2 = 1$ with the cylinder $y = 1 - x^2 - z^2$.

7. Find the tangent vector to the space curve $\mathbf{r}(t) = \langle \cos t, t^2, 2\sin t \rangle$ at the point $(1, 0, 0)$.

8. Calculate the speed of a object with position function $\mathbf{r}(t) = \langle \cos 3t, 4t, \sin 3t \rangle$.

9. An object with a mass of 2kg has position (in meters) after t seconds given by $\mathbf{r}(t) = \langle t^3, t, t^2 \rangle$. Calculate the force acting on the object when $t = 1$s.

10. Calculate the curvature of $\mathbf{r}(t) = \langle t^3, t, t^2 \rangle$ when $t = 1$.