Name: ____

Instructions: Answer all 9 problems. Show your work: even correct answers may receive little or no credit if a method of solution is not shown. Calculators, notes, cell phones, and other materials are not permitted.

Some useful formulas:

- The linearization of f near (x_0, y_0):

$$L(x,y) = f(x_0,y_0) + f_x(x_0,y_o)(x-x_0) + f_y(x_0,y_o)(y-y_0)$$

- An equation for the tangent plane to the level surface $F(x,y,z) = k$ at the point (x_0, y_0, z_0):

$$F_x(x_0,y_0,z_0)(x-x_0) + F_y(x_0,y_0,z_0)(y-y_0) + F_z(x_0,y_0,z_0)(z-z_0) = 0$$

- Second derivative test for a critical point (a,b) of $f(x,y)$: $D(a,b) = f_{xx}(a,b)f_{yy}(a,b) - [f_{xy}(a,b)]^2$.

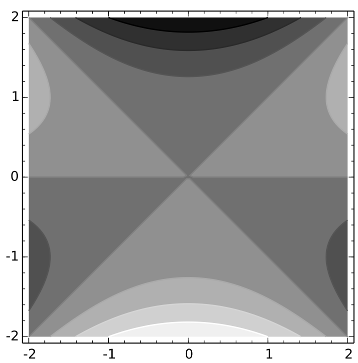
  If $D > 0$ and $f_{xx}(a,b) > 0$, then $f(a,b)$ is a local minimum;

  If $D > 0$ and $f_{xx}(a,b) < 0$, then $f(a,b)$ is a local maximum;

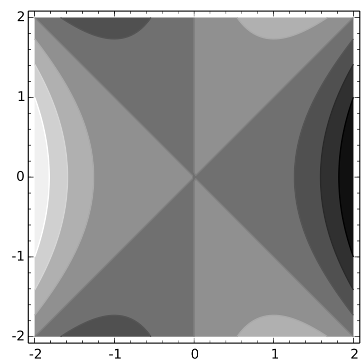
  If $D < 0$, then $f(a,b)$ is not a local extreme;

  If $D = 0$, then the test is inconclusive.

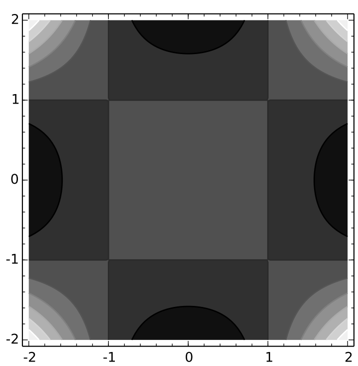
1. In the contour plots shown below lighter shades are higher, the x-axis is horizontal, the y-axis is vertical, and the point $(0,0)$ is in the center. Determine which plot is the contour plot of $z = xy^2 - x^3$. Explain your reasoning for possible partial credit.



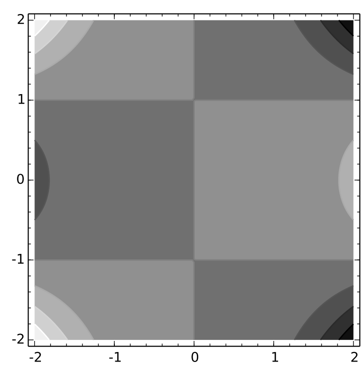
a)



b)



c)



d)

2. Explain why the limit does not exist: $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2 y}{x^3 + y^3}$.

3. Find $\frac{\partial z}{\partial x}$ when $yz + e^{xy} = z^2$.

4. Find an equation for the plane tangent to the surface $z = y\sin\left(\frac{x}{y}\right)$ at the point $(\pi, 6, 3)$.

5. Find $\frac{\partial w}{\partial r}$ if $w = xy + xz + yz$, $x = r\cos\theta$, $y = r\sin\theta$, and $z = r^2$.

6. Find the maximum rate of change of the function $f(x, y) = \sqrt{x + 2y}$ at the point $(2, 1)$ and the direction in which it occurs (the direction does not need to be a unit vector).

7. Calculate $D_{\mathbf{u}}f(-1, 1)$ for $f(x, y) = 2xy^2 + x$ and $\mathbf{u} = \left\langle \frac{1}{2}, \frac{\sqrt{3}}{2} \right\rangle$.

8. Find all the critical points of $f(x, y) = x^2 + y + 4xy + 1$ and determine if each is a local minimum, local maximum, or neither.

9. Find the location of the minimum value of $f(x, y, z) = x^2 + y^2 + z^2$ given the constraint $x + 2y + 3z = 21$.