Name: ____

Instructions: Answer all 7 problems. Show your work: even correct answers may receive little or no credit if a method of solution is not shown. Calculators, notes, cell phones, and other materials are not permitted.

Some useful formulas:

- Spherical–Cartesian conversions: $x = \rho \sin\phi \cos\theta$, $y = \rho \sin\phi \sin\theta$, $z = \rho\cos\phi$, and $x^2 + y^2 + z^2 = \rho^2$.
- The Jacobian of the transformation $x = g(u, v)$, $y = h(u, v)$ is $\dfrac{\partial(x, y)}{\partial(u, v)} = \left(\dfrac{\partial x}{\partial u}\right)\left(\dfrac{\partial y}{\partial v}\right) - \left(\dfrac{\partial x}{\partial v}\right)\left(\dfrac{\partial y}{\partial u}\right)$.

1. Evaluate the definite integral $\iint_R 8x \, dA$ where R is the region bounded by the lines $y = 2$, $y = 2x$, and $y = -x$.

2. Evaluate the integral by first reversing the order of integration: $\int_0^1 \int_{\sqrt{y}}^1 e^{x^3}\, dx dy$.

3. Find an integral expression for the moment M_y of the lamina with density $\rho(x, y) = x^2 + y^2$ occupying the polar region $R = \{(r, \theta) : 0 \leq r \leq 1 + \cos\theta, 0 \leq \theta \leq \pi\}$. Do not evaluate the integral.

4. Evaluate the integral $\iiint_E x^2 dV$ where E is the region bounded by the xy-plane, the plane $z = y$, and the surface $y = 1 - x^2$.

5. Find an integral expression in spherical coordinates for the moment M_{xz} of the solid inside the sphere $x^2 + y^2 + z^2 = 1$ and between the planes $y = \frac{x}{\sqrt{3}}$ and $y = \sqrt{3}x$ and with $x \geq 0$ and $y \geq 0$. Do not evaluate the integral. Hint: $\tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}}$ and $\tan\left(\frac{\pi}{3}\right) = \sqrt{3}$.

6. Express the integral $\int_0^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} \int_0^{\sqrt{4-x^2-y^2}} (x^2 + y^2)\, dz\, dx\, dy$ as an integral in cylindrical coordinates. Do not evaluate the integral.

7. The transformation $x = u - \sqrt{3}v$, $y = \sqrt{3}u + v$ transforms the unit square $S = \{(u, v) : 0 \leq u \leq 1,\ 0 \leq v \leq 1\}$ into the diamond-shaped region D bounded by the lines $y = \sqrt{3}x$, $y = \sqrt{3}x + 4$, $x = -\sqrt{3}y$, and $x = -\sqrt{3}y + 4$. Use this transformation to evaluate the integral $\iint_D x + \sqrt{3}y\, dA$.