Name:

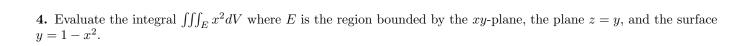
INSTRUCTIONS: Answer all 7 problems. Show your work: even correct answers may receive little or no credit if a method of solution is not shown. Calculators, notes, cell phones, and other materials are not permitted.

Some useful formulas:

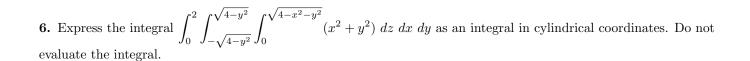
- Spherical–Cartesian conversions: $x = \rho \sin \phi \cos \theta$, $y = \rho \sin \phi \sin \theta$, $z = \rho \cos \phi$, and $x^2 + y^2 + z^2 = \rho^2$.
- The Jacobian of the transformation $x=g(u,v),\,y=h(u,v)$ is $\frac{\partial(x,y)}{\partial(u,v)}=\left(\frac{\partial x}{\partial u}\right)\left(\frac{\partial y}{\partial v}\right)-\left(\frac{\partial x}{\partial v}\right)\left(\frac{\partial y}{\partial u}\right).$
- 1. Evaluate the definite integral $\iint_R 8x \ dA$ where R is the region bounded by the lines y=2, y=2x, and y=-x.

2. Evaluate the integral by first reversing the order of integration: $\int_0^1 \int_{\sqrt{y}}^1 e^{x^3} dx dy$.

3. Find an integral expression for the moment M_y of the lamina with density $\rho(x,y)=x^2+y^2$ occupying the polar region $R=\{(r,\theta): 0\leq r\leq 1+\cos\theta, 0\leq \theta\leq\pi\}$. Do not evaluate the integral.



5. Find an integral expression in spherical coordinates for the moment M_{xz} of the solid inside the sphere $x^2+y^2+z^2=1$ and between the planes $y=\frac{x}{\sqrt{3}}$ and $y=\sqrt{3}x$ and with $x\geq 0$ and $y\geq 0$. Do not evaluate the integral. Hint: $\tan\left(\frac{\pi}{6}\right)=\frac{1}{\sqrt{3}}$ and $\tan\left(\frac{\pi}{3}\right)=\sqrt{3}$.



7. The transformation $x=u-\sqrt{3}v,\ y=\sqrt{3}u+v$ transforms the unit square $S=\{(u,v):0\leq u\leq 1,\ 0\leq v\leq 1\}$ into the diamond-shaped region D bounded by the lines $y=\sqrt{3}x,\ y=\sqrt{3}x+4,\ x=-\sqrt{3}y,\ \text{and}\ x=-\sqrt{3}y+4.$ Use this transformation to evaluate the integral $\iint_D x+\sqrt{3}y\ dA$.