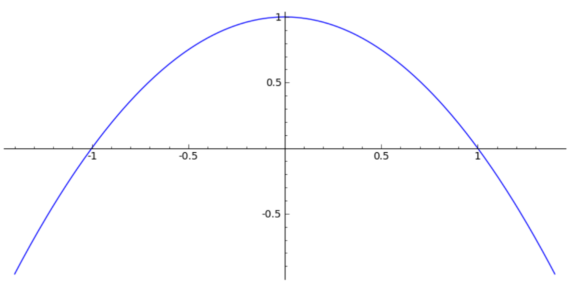
INSTRUCTIONS: Answer all 10 problems. Show all your work: even correct answers may receive little or no credit if a method of solution is not shown. Calculators, notes, cell phones, and other materials are not permitted.

NAME. ____________________________________

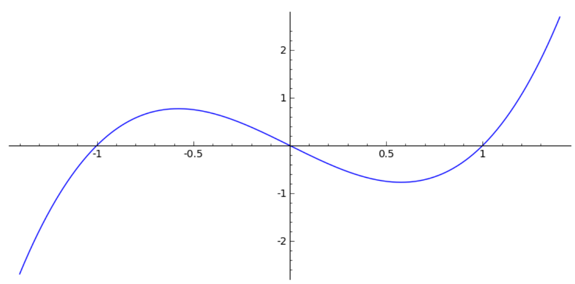
You may find the following helpful:

- Half-angle formulas: $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$ and $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$;
- Derivative of a polar curve: $\dfrac{dy}{dx} = \dfrac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$;
- Arc length of a polar curve: $L = \displaystyle\int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$;
- Conic sections with foci at the origin and directrices parallel to an axis: $r = \dfrac{ed}{1 \pm e\cos\theta}$ or $r = \dfrac{ed}{1 \pm e\sin\theta}$;
- Some values of $\tan\theta$: $\tan 0 = 0$, $\tan(\frac{\pi}{6}) = \frac{1}{\sqrt{3}}$, $\tan(\frac{\pi}{4}) = 1$, $\tan(\frac{\pi}{3}) = \sqrt{3}$, and $\tan(\frac{\pi}{2})$ is undefined.

1. Use the following graphs of $x = f(t)$ and $y = g(t)$ to sketch the parametric curve $x = f(t)$, $y = g(t)$. Indicate the direction in which the curve is traced as t increases and give coordinates for axis intercepts.



$x = f(t)$



$y = g(t)$

2. Eliminate the parameter to find a Cartesian equation for the curve with parametric equations $x = e^t - 1$, $y = e^{2t}$.

3. Find an equation for the line tangent to the parametric curve $x(t) = t^3 - 1$, $y(t) = 1 + \ln t$ at the point $(x, y) = (0, 1)$.

4. Find $\dfrac{d^2y}{dx^2}$ for $x(t) = 1 + t^2$, $y(t) = t + t^2$.

5. Find the area enclosed between the parametric curve $x = t - t^2$, $y = 2\sqrt{t}$ and the y-axis.

6. Sketch the polar curve $r = 1 + \cos(2\theta)$. Mark the axis intercepts clearly.

7. Find the slope of the line tangent to the polar curve $r = 2 + 3\sin(2\theta)$ when $\theta = \pi$.

8. Determine the area of the region that lies inside the curve $r = 1$ and outside the curve $r = \cos(2\theta)$.

9. Calculate the arc length of the polar curve $r = \theta^2$ for $1 \leq \theta \leq 2$.

10. Determine if the conic section defined by $r = \dfrac{-5}{\cos\theta - 2}$ is an ellipse, a parabola, or a hyperbola and find a Cartesian equation for its directrix.