Name(s):
Math 259 | Partial Derivatives | March 6, 2015

1. Suppose $\frac{d}{du}[f(u)] = e^{u^2}$.

a) Find $\dfrac{\partial}{\partial x}\left[f\left(\dfrac{x}{y}\right)\right]$

b) Find $\dfrac{\partial}{\partial y}\left[f\left(\dfrac{x}{y}\right)\right]$

2. We can keep differentiating to find second partial derivatives, third partial derivatives, and so on. Pay careful attention to the order of differentiation. The second partial derivatives are:

$$f_{xx} = \frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) \qquad f_{yy} = \frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right)$$

$$f_{xy} = \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) \qquad f_{yx} = \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right)$$

Calculate the second partial derivatives of $f\left(\frac{x}{y}\right)$ from problem 1.

3. Clairaut's Theorem tells us that the mixed second partial derivatives are always equal (provided they are both continuous). Verify that $\dfrac{\partial^2}{\partial y \partial x}\left[f\left(\dfrac{x}{y}\right)\right] = \dfrac{\partial^2}{\partial x \partial y}\left[f\left(\dfrac{x}{y}\right)\right]$.

4. In the contour plot of $z = g(x, y)$ shown below lighter areas are higher and darker areas are lower. Use the contour plot determine the sign of each of the partial derivatives.

a) $g_x(0.5, 1)$

b) $g_y(0.5, 1)$

c) $g_y(-2, 0.5)$

d) $g_x(0, 0)$