Name(s):

Definiton. A point (a, b) is a *critical point* of f if any of the following is true:

- $f_x(a, b) = 0$ and $f_y(a, b) = 0$;
- $f_x(a, b)$ is undefined;
- $f_y(a, b)$ is undefined.

Theorem. *If f has a local maximum or local minimum at (a, b), then (a, b) is a critical point of f.*

Not every critical point is a local maximum or minimum. The second derivative test gives us a way of determining if a critical point is a local minimum or maximum.

Theorem (Second Derivative Test)**.** *Suppose the second partial derivatives of f are continuous on a disk with center (a, b), and suppose that $f_x(a, b) = 0$ and $f_y(a, b) = 0$. Let*

$$D = f_{xx}(a, b)f_{yy}(a, b) - [f_{xy}(a, b)]^2 .$$

a) If $D > 0$ and $f_{xx}(a, b) > 0$, then $f(a, b)$ is a local minimum.

b) If $D > 0$ and $f_{xx}(a, b) < 0$, then $f(a, b)$ is a local maximum.

c) If $D < 0$, then $f(a, b)$ is not a local minimum or a local maximum.

d) If $D = 0$, then the test is inconclusive.

1. Find all the local extremes of $f(x, y) = xy - 2x - 2y - x^2 - y^2$.

2. Find all the local extremes of $f(x, y) = xe^{-2x^2-2y^2}$.

3. Find the absolute maximum and minumum of $f(x, y) = xe^{-2x^2-2y^2}$ over the disk $x^2 + y^2 \leq 1$.