1. A solid occupies the region above the plane $z = 1$, below the plane $z = y$, and inside the cylinder $x^2 + y^2 = 4$.

a) Express the volume of the solid as an triple integral in Cartesian coordinates.

Solution. $V = \int_{-\sqrt{3}}^{\sqrt{3}} \int_{1}^{\sqrt{4-x^2}} \int_{1}^{\sqrt{y}} dzdydx$ (other orders of integration are possible).

b) Express the volume of the solid as a triple integral in cylindrical coordinates. Hint: $\tan \frac{\pi}{6} = \frac{1}{\sqrt{3}}$.

Solution. $V = \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \int_{\frac{1}{\sin\theta}}^{2} \int_{1}^{r\sin\theta} r dzdrd\theta$ (other orders of integration are possible).

c) Calculate the volume of the solid. It may be helpful to recall that $\int \csc^2 u \; du = -\cot u + C$.

Solution. Evaluating either the Cartesian or cylindrical integral gives $V = 3\sqrt{3} - \frac{4}{3}\pi$.

2. A solid (the "sno-cone") occupies the region above $z = \sqrt{x^2+y^2}$ and below the sphere $x^2 + y^2 + z^2 = 4$.

a) Express the volume of the solid as an integral in cylindrical coordinates.

Solution. $V = \int_{0}^{2\pi} \int_{0}^{\sqrt{2}} \int_{r}^{\sqrt{4-r^2}} r dzdrd\theta$ (other orders of integration are possible).

b) Express the volume of the solid as an integral in spherical coordinates.

Solution. $V = \int_{0}^{\frac{\pi}{4}} \int_{0}^{2\pi} \int_{0}^{2} \rho^2 \sin\phi d\rho d\theta d\phi$ (other orders of integration are possible).

c) Calculate the volume of the solid.

Solution. Evaluating either the cylindrical or spherical integral gives $V = \frac{16}{3}\pi\left(1 - \frac{1}{\sqrt{2}}\right)$.