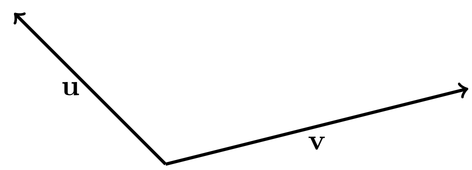
Chapter 11

1. Vectors $\mathbf{u}$ and $\mathbf{v}$ are shown. Draw the indicated vectors on the figure (as accurately as you can manage). Label your vectors clearly.

a) $\mathbf{u}+\mathbf{v}$

b) $\mathbf{proj_v u}$



2. Find the work done by a force $\mathbf{F} = \langle 2, -1, 1\rangle$ that moves an object from $(1, 0, 1)$ to $(1, -3, 5)$ along a straight line (with measurements in Newtons and meters, respectively).

3. Find a unit vector perpendicular to both of the vectors $\langle 2, 0, 2\rangle$, and $\langle -1, 1, 1\rangle$.

4. Let $\mathbf{a} = \langle 1, -2, 3\rangle$ and $\mathbf{b} = \langle 1, 4, -2\rangle$. Calculate $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{a})$.

5. Let θ be the angle between the lines $\mathbf{r}_1(t) = \langle t + 1, -t, t\rangle$ and $\mathbf{r}_2(t) = \langle 2t, t - 2, 2t - 1\rangle$. Calculate $\cos\theta$. (Technically, there are two angles between the lines; cosine of either angle is an acceptable answer).

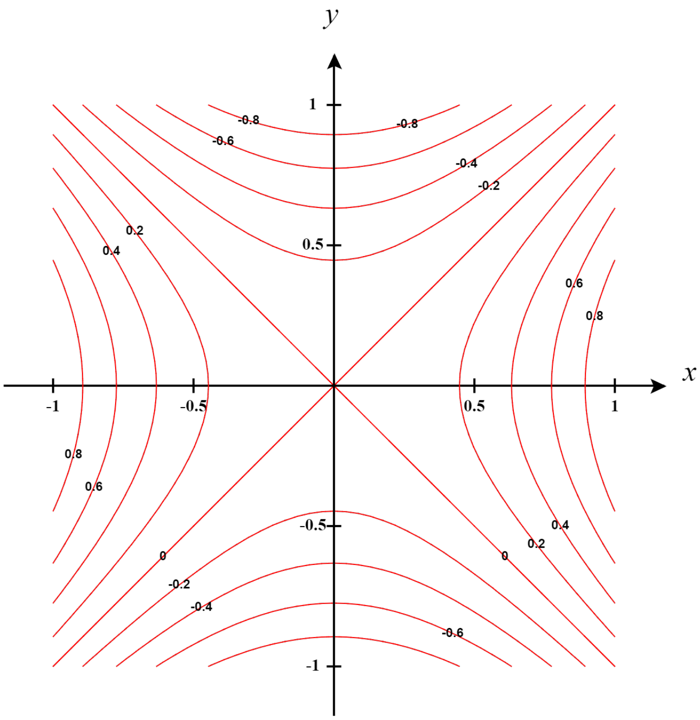
6. Determine the distance from the point $(-2, 2, 1)$ to the line $\mathbf{r}(t) = \langle 1 + t, -t, -1\rangle$

7. Find an equation for the line the tangent to the curve $\mathbf{r}\,(t) = \langle 1 + t^2, t, \sin t\rangle$ at the point $(1, 0, 0)$.

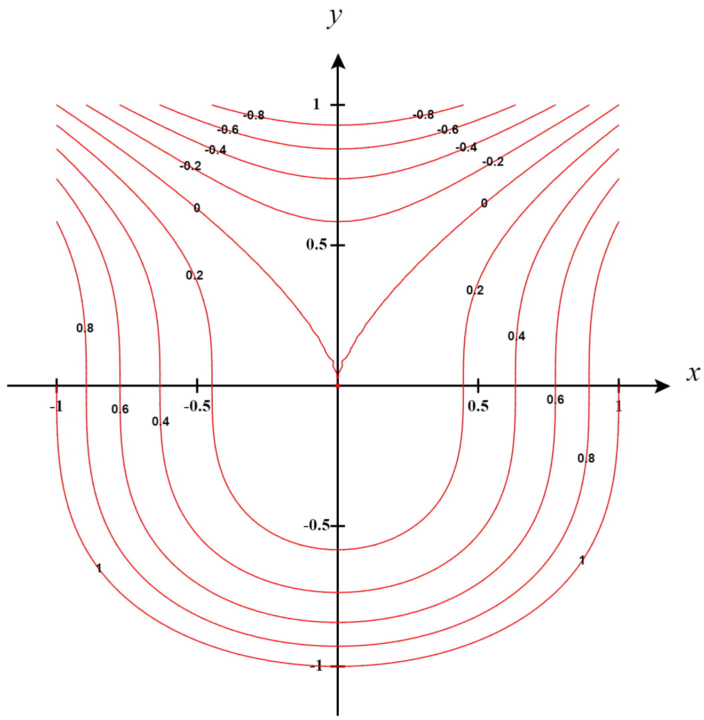
8. Find an integral giving the length of the curve $\mathbf{r}\,(t) = \langle 2t, 2t^2, \cos(2t)\rangle$ for $0 \le t \le 3$. Do not evaluate the integral.

9. An object has initial velocity $\mathbf{v}(0) = \langle 1, 0, 1\rangle$ m/s and is subject to a force that produces an acceleration of $\mathbf{a}(t) = \langle 6t, \cos t, 1\rangle$ m/s^2. Find a vector function for the velocity of the object.

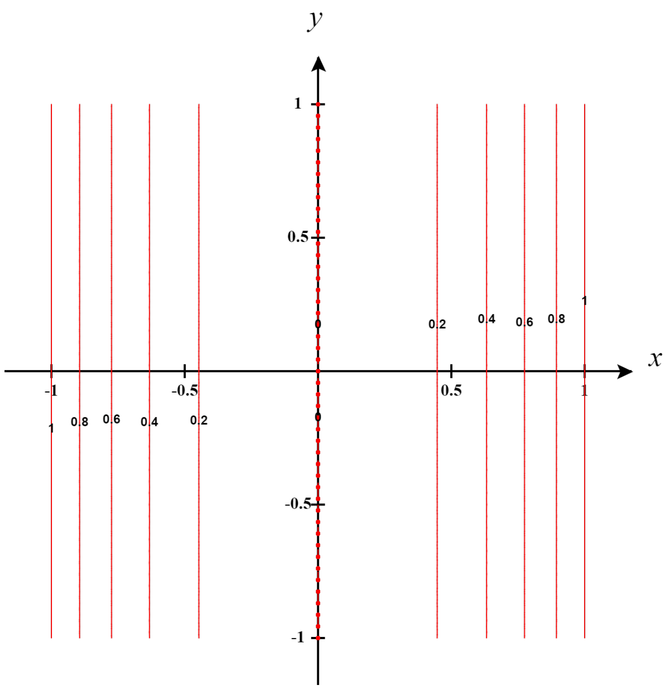
10. Match the contour plot to the function. Explain your reasoning for possible partial credit.



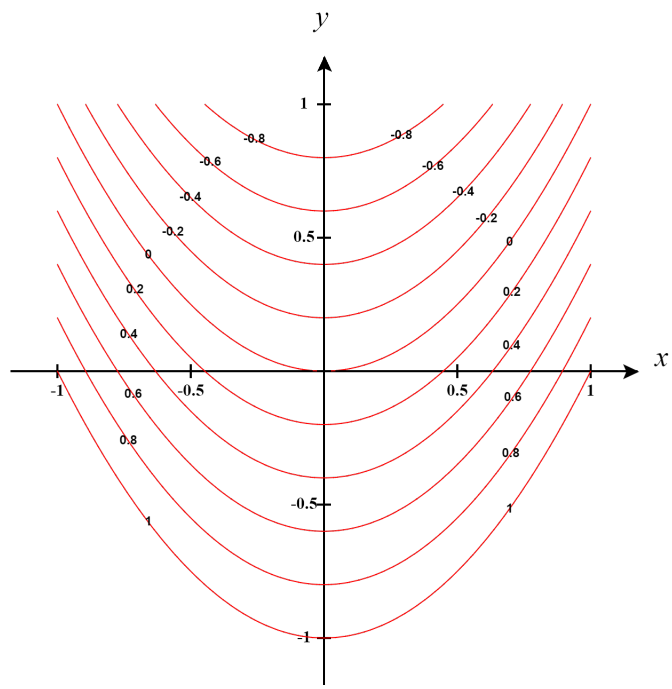
a)



b)



c)



d)

i) $f(x,y) = x^2$ ii) $f(x,y) = x^2 - y$ iii) $f(x,y) = x^2 - y^2$ iv) $f(x,y) = x^2 - y^3$

11. Find the limit or use the two-path test to explain why the limit does not exist: $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x+y^3}{x^2+y^2}$

12. Determine if the planes $x + y + 2z = 4$ and $3x + y - z = 3$ are parallel or intersecting. If they intersect, find a point on the line of intersection.

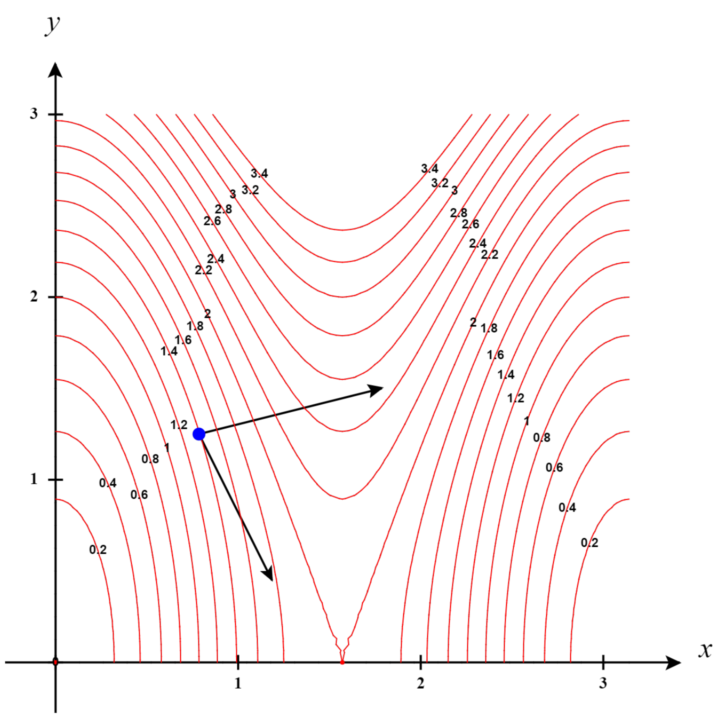
13. Find an equation for the plane tangent to the surface $xy \sin z = 1$ at the point $(1, 2, \frac{\pi}{6})$.

14. Find $\frac{\partial z}{\partial t}$ if $z = \frac{x}{y}$, $x = ts$ and $y = t^2 - s$.

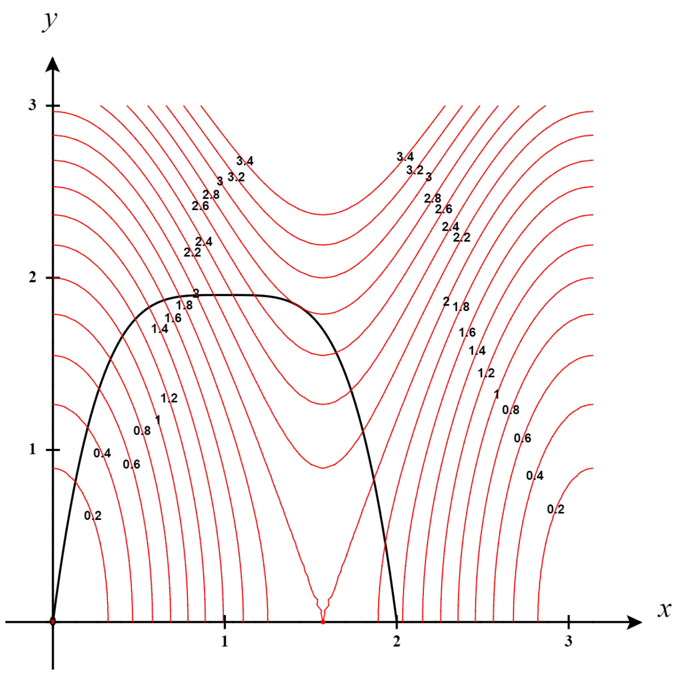
15. Calculate $D_{\mathbf{u}} f(1, 2)$ for $f(x, y) = 3x^2 + y^2$ and $\mathbf{u} = \langle \frac{3}{5}, \frac{4}{5} \rangle$.

16. Find all the critical points of the function $f(x, y) = x^4 + 2y^2 - 4xy$ and use the second derivative test to determine if each is a local minimum, local maximum, saddle, or indeterminate.

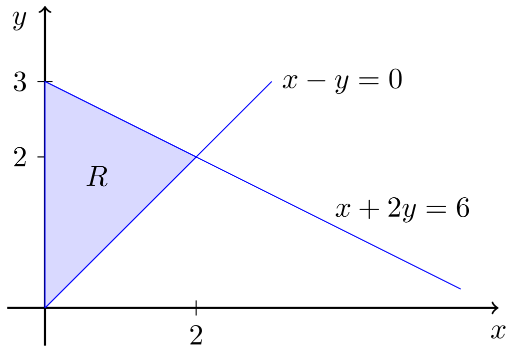
17. The vectors $\mathbf{u} = \langle 1, 0.25 \rangle$ and $\mathbf{v} = \langle 0.4, -0.8 \rangle$, both anchored at the point $(\frac{\pi}{4}, 1.25)$, are shown in the contour plot of a function $f(x, y)$. Determine which directional derivative $D_{\frac{\mathbf{u}}{|\mathbf{u}|}} f\left(\frac{\pi}{4}, 1.25\right)$ or $D_{\frac{\mathbf{v}}{|\mathbf{v}|}} f\left(\frac{\pi}{4}, 1.25\right)$ is greater.



18. A contour plot for $z = f(x, y)$ is shown together with a constraint curve $g(x, y) = c$. use the plot to estimate the maximum value of $f(x, y)$ given the constraint $g(x, y) = c$.



19. Evaluate the integral $\iint_R 2x \; dA$ where R is the region shown below.



20. Evaluate the integral (change the order of integration if necessary): $\int_0^4 \int_{x/2}^2 \cos(y^2) \; dy \; dx$

21. Evaluate the integral (consider using polar coordinates): $\int_0^{\sqrt{2}} \int_y^{\sqrt{4-y^2}} \frac{1}{1+x^2+y^2} \; dx \; dy$

22. Find the volume of the solid bounded between the paraboloids $z = x^2 + y^2$ and $z = 2 - x^2 - y^2$.

23. Let $E = \{(x, y, z) : 0 \le y \le x \le \pi, \; 0 \le z \le 2\}$. Express $\iiint_E f(x, y, z) \; dV$ as an iterated integral (do not evaluate the integral).

24. Set up an iterated integral giving the volume of the solid occupying the region above the cone $\varphi = \pi/3$ and inside the sphere $\rho = 4\cos\varphi$ (a sphere of radius 2 with center at $(x, y, z) = (0, 0, 2)$). You do not need to evaluate the integral.

25. Rewrite the integral in spherical coordinates: $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} (x^2 + y^2 + z^2)^2 \; dz \; dy \; dx$

26. Which of the following integrals are equal to $\int_0^1 \int_0^{x^2} \int_0^y f(x, y, z) \; dz \; dy \; dx$ (that is, which are integrals over the same region of space)? Mark all correct answers.

a) $\int_0^y \int_0^{x^2} \int_0^1 f(x, y, z) \; dx \; dy \; dz$

b) $\int_0^1 \int_z^1 \int_{\sqrt{y}}^1 f(x, y, z) \; dx \; dy \; dz$

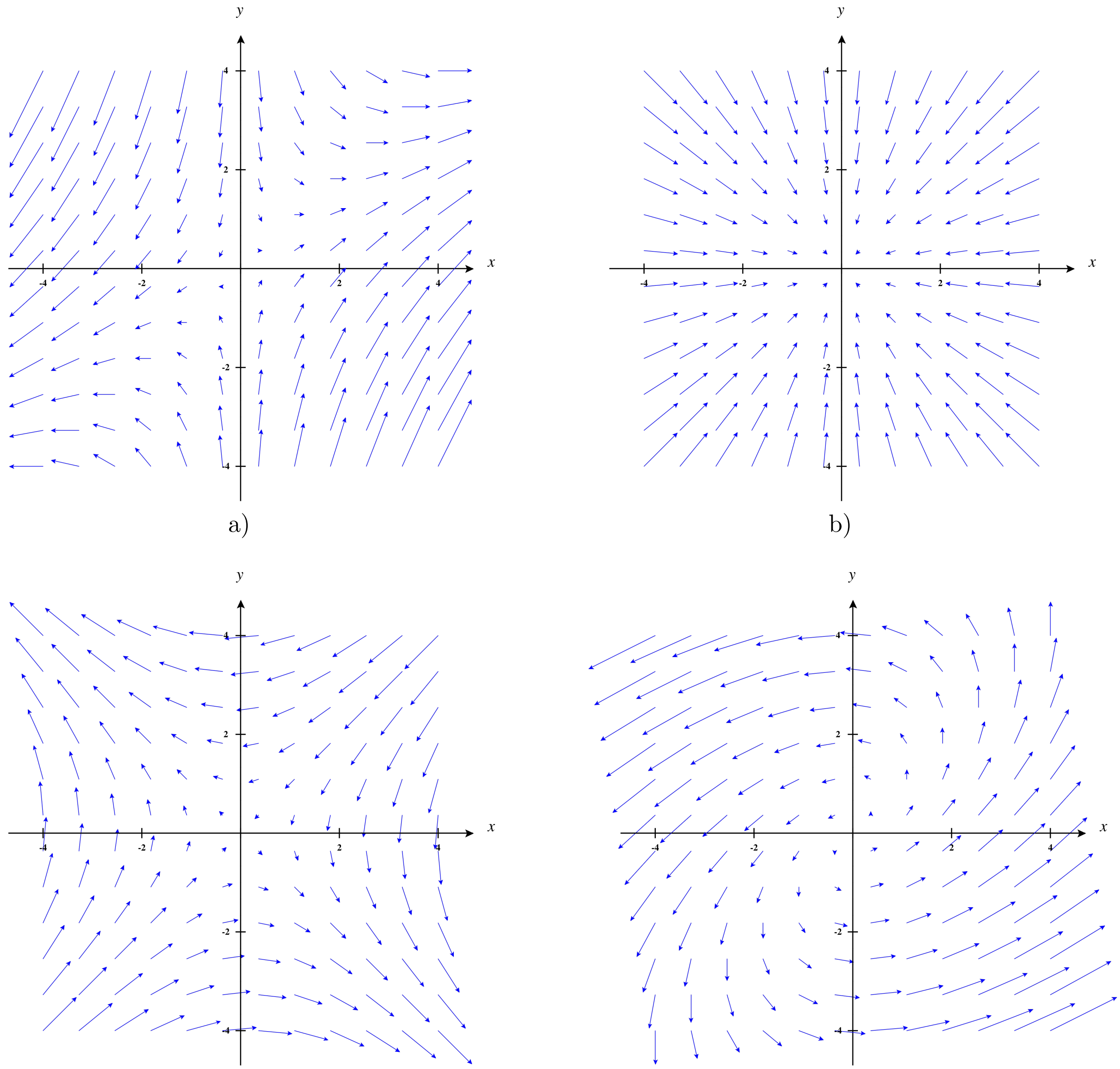
c) $\int_0^1 \int_0^z \int_{\sqrt{y}}^1 f(x, y, z) \; dx \; dy \; dz$

d) $\int_0^1 \int_{\sqrt{y}}^1 \int_0^y f(x, y, z) \; dz \; dx \; dy$

e) $\int_0^1 \int_0^{x^2} \int_z^{x^2} f(x, y, z) \; dy \; dz \; dx$

27. Evaluate the line integral $\int_C x\ ds$ where C is the arc of the parabola $y = x^2$ from $(0,0)$ to $(1,1)$.

28. Match the vector field to its graph. Explain your reasoning for possible partial credit.



a) b) c) d)

i) $\mathbf{F}(x,y) = \langle -x, -y\rangle$ ii) $\mathbf{F}(x,y) = \langle -y, -x\rangle$ iii) $\mathbf{F}(x,y) = \langle x, x-y\rangle$ iv) $\mathbf{F}(x,y) = \langle x-y, x\rangle$

29. Find a potential function for the conservative vector field $\mathbf{F} = \langle y+z, x+z, x+y\rangle$.

30. Evaluate the line integral $\displaystyle\int_C \mathbf{F}\cdot d\mathbf{r}$ for $\mathbf{F} = \langle y, x+2y\rangle$ and $C : \mathbf{r}(t) = \langle \sin t\cos t, \cos^2 t\rangle$ for $0 \le t \le \pi$. Hint: is the vector field conservative?

31. Let $\mathbf{F} = \langle x, y, z\rangle$ and let S be the surface of the paraboloid $z = 4 - x^2 - y^2$ over the square $0 \le x \le 1$ and $0 \le y \le 1$. Calculate $\iint_S \mathbf{F}\cdot\mathbf{n}\ dS$.

32. Let $\mathbf{F} = \langle xe^{yz}, ye^{xz}, ze^{xy} \rangle$ and let S be the part of the sphere $x^2 + y^2 + z^2 = 4$ with $z \geq 0$. Use Stokes' theorem to calculate $\iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, dS$.

33. Let $\mathbf{F} = \langle x^2y, -x^2z, yz^2 \rangle$ and let S be the surface of the rectangular box with $0 \leq x \leq 2$, $0 \leq y \leq 2$ and $0 \leq z \leq 1$. Use the divergence theorem to calculate the outward flux of $\mathbf{F}$ through S.