Basics:	$\mathbf{a} = \langle a_1, a_2, a_3 \rangle$	$\mathbf{b} = \langle b_1, b_2, b_3 \rangle$	$\lvert\mathbf{a}\rvert = \sqrt{a_1^2 + a_2^2 + a_3^2}$
Dot products:	$\mathbf{proj_b a} = \left(\dfrac{\mathbf{a}\cdot\mathbf{b}}{\mathbf{b}\cdot\mathbf{b}}\right)\mathbf{b}$	$\text{comp}_{\mathbf{b}}\mathbf{a} = \dfrac{\mathbf{a}\cdot\mathbf{b}}{\lvert\mathbf{b}\rvert}$	$\mathbf{a}\cdot\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3 = \lvert\mathbf{a}\rvert\,\lvert\mathbf{b}\rvert\cos\theta$
Cross products:	$\lvert\mathbf{a}\times\mathbf{b}\rvert = \lvert\mathbf{a}\rvert\,\lvert\mathbf{b}\rvert\sin\theta$	$\mathbf{a}\times\mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$	$\mathbf{a}\times\mathbf{b} = \langle a_2b_3 - a_3b_2, a_3b_1 - a_1b_3, a_1b_2 - a_2b_1 \rangle$
Applications:	$W = \mathbf{F}\cdot\mathbf{d}$	$\tau = \mathbf{r}\times\mathbf{F}$	Length: $L = \int_a^b \lvert\mathbf{r}'(t)\rvert\,dt$
Unit vectors:	$\mathbf{T}(t) = \dfrac{\mathbf{r}'(t)}{\lvert\mathbf{r}'(t)\rvert}$	$\mathbf{N}(t) = \dfrac{\mathbf{T}'(t)}{\lvert\mathbf{T}'(t)\rvert}$	$\mathbf{B}(t) = \mathbf{T}(t)\times\mathbf{N}(t)$

- Gradient vector of $F(x, y, z)$:

$$\nabla F(x,y,z) = \langle F_x(x,y,z), F_y(x,y,z), F_z(x,y,z) \rangle$$

- Directional derivative of $f(x, y)$ in the direction of unit vector $\mathbf{u}$ at (a, b):

$$D_{\mathbf{u}} f(a,b) = \nabla f(a,b)\cdot\mathbf{u}$$

- The plane tangent to $F(x, y, z) = k$ at the point (a, b, c):

$$\nabla F(a,b,c)\cdot\langle x-a, y-b, z-c\rangle = 0$$

- The plane tangent to $z = f(x, y)$ at (a, b) (linearization of f):

$$z = f_x(a,b)(x-a) + f_y(a,b)(y-b) + f(a,b)$$

- Differential of $z = f(x, y)$ at (a, b):

$$\Delta z \approx dz = f_x(a,b)dx + f_y(a,b)dy$$

- Second derivative test for a critical point (a, b) of $f(x, y)$: $D(a,b) = f_{xx}(a,b)f_{yy}(a,b) - [f_{xy}(a,b)]^2$.
  If $D > 0$ and $f_{xx}(a,b) > 0$, then $f(a,b)$ is a local minimum;
  If $D > 0$ and $f_{xx}(a,b) < 0$, then $f(a,b)$ is a local maximum;
  If $D < 0$, then (a,b) is a saddle point;
  If $D = 0$, then the test is inconclusive.

Applications	Translations		Differentials
$A = \iint_R dA$	Rectangular $\to$ Cylindrical	Rectangular $\to$ Spherical	$dV = \dots$
$V = \iint_R f(x,y)\ dA$	$x = r\cos\theta$	$x = \rho\sin\varphi\cos\theta$	$dx\ dy\ dz$
$V = \iiint_S dV$	$y = r\sin\theta$	$y = \rho\sin\varphi\sin\theta$	$r\ dz\ dr\ d\theta$
$m = \iiint_S f(x,y,z)\ dV$	$z = z$	$z = \rho\cos\varphi$	$\rho^2\sin\varphi\ d\rho\ d\varphi\ d\theta$
	$x^2 + y^2 = r^2$	$x^2 + y^2 + z^2 = \rho^2$	

Definitions

Curl (3-d): $\text{curl}\mathbf{F} = \nabla \times \mathbf{F} = \left\langle \frac{\partial h}{\partial y} - \frac{\partial g}{\partial z}, \frac{\partial f}{\partial z} - \frac{\partial h}{\partial x}, \frac{\partial g}{\partial x} - \frac{\partial f}{\partial y} \right\rangle$

Curl (2-d): $\text{curl}\mathbf{F} = \frac{\partial g}{\partial x} - \frac{\partial f}{\partial y}$

Divergence (3-d): $\text{div}\mathbf{F} = \nabla \cdot \mathbf{F} = \frac{\partial f}{\partial x} + \frac{\partial g}{\partial y} + \frac{\partial h}{\partial z}$

Divergence (2-d): $\text{div}\mathbf{F} = \nabla \cdot \mathbf{F} = \frac{\partial f}{\partial x} + \frac{\partial g}{\partial y}$

Theorems

Green's (general): $\oint_C P dx + Q dy = \iint_R \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\ dA$

Green's (circulation): $\oint_C \mathbf{F} \cdot d\mathbf{r} = \iint_R \frac{\partial g}{\partial x} - \frac{\partial f}{\partial y}\ dA$

Green's (flux): $\oint_C \mathbf{F} \cdot \mathbf{n}\ ds = \iint_R \frac{\partial f}{\partial x} + \frac{\partial g}{\partial y}\ dA$

Stokes': $\oint \mathbf{F} \cdot d\mathbf{r} = \iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\ dS$

Divergence: $\iint_S \mathbf{F} \cdot \mathbf{n}\ dS = \iiint_D \nabla \cdot \mathbf{F}\ dV$

Test for conservative v.f.: $\mathbf{F}$ is conservative if and only if curl$\mathbf{F} = 0$

Fundamental theorem of line integrals: if $\mathbf{F}$ is conservative, then $\int_C \mathbf{F} \cdot d\mathbf{r} = \varphi(B) - \varphi(A)$

Line integrals

Scalar: $\int_C f\ ds = \int_a^b f(\mathbf{r}(t)) \left|\mathbf{r}'(t)\right| dt$

Circulation/work: $\int_C \mathbf{F} \cdot d\mathbf{r} = \int_C f dx + g dy = \int_a^b \mathbf{F}(\mathbf{r}(t)) \cdot \mathbf{r}'(t)\ dt$

Flux: $\int_C \mathbf{F} \cdot \mathbf{n}\ ds = \int_C f dy - g dx = \int_a^b \left[f(\mathbf{r}(t))y'(t) - g(\mathbf{r}(t))x'(t)\right]\ dt$

Surface integrals

Scalar: $\iint_S f\ dS = \iint_R f(\mathbf{r}(u,v)) \left|\mathbf{t}_u \times \mathbf{t}_v\right|\ dA$

Scalar for $z = z(x,y)$: $\iint_S f\ dS = \iint_R f(x, y, z(x,y)) \sqrt{z_x^2 + z_y^2 + 1}\ dA$

Flux: $\iint_S \mathbf{F} \cdot \mathbf{n}\ dS = \int_R \mathbf{F} \cdot (\mathbf{t}_u \times \mathbf{t}_v)\ dA$

Flux for $z = z(x,y)$: $\iint_S \mathbf{F} \cdot \mathbf{n}\ dS = \iint_R \mathbf{F} \cdot \langle -z_x, -z_y, 1\rangle\ dA$