VECTOR PRODUCTS

Definition. The **dot product** of vectors $\mathbf{u}$ and $\mathbf{v}$ is $\boxed{\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}|\,|\mathbf{v}| \cos\theta}$ where θ is the angle between $\mathbf{u}$ and $\mathbf{v}$.

Often this definition is used in conjunction with the following formula for the dot product. Note that I've stated only the 3-d version; I think you'll be able to figure out the 2-d version.

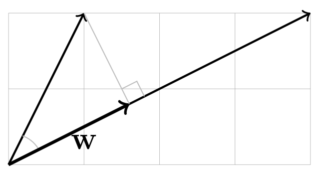
Theorem. *If* $\mathbf{u} = \langle u_1, u_2, u_3 \rangle$ *and* $\mathbf{v} = \langle v_1, v_2, v_3 \rangle$, *then* $\boxed{\mathbf{u} \cdot \mathbf{v} = u_1 v_1 + u_2 v_2 + u_3 v_3}$

1. Let $\mathbf{u} = \langle 1, 2 \rangle$ and let $\mathbf{v} = \langle 4, 2 \rangle$.

a) Calculate $\mathbf{u} \cdot \mathbf{v}$

b) Calculate $|\mathbf{u}|$ and $|\mathbf{v}|$

c) Calculate $\cos\theta$



2. Shown above is a diagram of the vectors from problem 1 (you may want to fill in the missing labels). The vector $\mathbf{w}$ is the **orthogonal projection of u onto v**.

a) Calculate $|\mathbf{w}|$

b) Find a unit vector with the same direction as $\mathbf{w}$

c) Find the vector components of $\mathbf{w}$

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3. You probably already know that work is force times distance. However, this only applies when the force and the displacement are in the same direction. The more general version is $\boxed{\mathbf{W} = \mathbf{F} \cdot \mathbf{d}}$ where work, force, and displacement are all vectors. Use this to calculate the work done by a ski lift that moves 200 kg of skiers along the vector $\langle 80, 350, 300\rangle$ (measured in meters) while subject to the force of gravity and wind exerting a force of $\langle 15, -120, -60\rangle$ Newtons.

Definition. If $\mathbf{u} = \langle u_1, u_2, u_3\rangle$ and $\mathbf{v} = \langle v_1, v_2, v_3\rangle$, then the **cross product** of $\mathbf{u}$ and $\mathbf{v}$ is

$$\boxed{\mathbf{u} \times \mathbf{v} = \langle u_2v_3 - u_3v_2,\ u_3v_1 - u_1v_3,\ u_1v_2 - u_2v_1\rangle}$$

(Note that the cross product is only defined for 3-dimensional vectors.)

4. The coordinate vectors of $\mathbb{R}^3$ are $\mathbf{i} = \langle 1, 0, 0\rangle$, $\mathbf{j} = \langle 0, 1, 0\rangle$, and $\mathbf{k} = \langle 0, 0, 1\rangle$.

a) Find $\mathbf{i} \times \mathbf{j}$

b) Find $\mathbf{j} \times \mathbf{i}$

c) Find $\mathbf{j} \times \mathbf{k}$

d) Find two unit vectors whose cross product is $\mathbf{j}$